

410

مسألة نكامل

في منتديات الرياضيات

حل وترتيب

الاستاذ / حسين فاضل الخعاسى

قامت بترجمتها لكم

مكتبة معلمي الرياضيات ١

تابعونا على:

<https://www.facebook.com/groups/mathtop4>

<http://ahmadybooks.blogspot.com>

$$1) \int \frac{dx}{\sin x \cos x}$$

$$\begin{aligned} &= \int \frac{2}{\sin 2x} dx = \int \csc 2x \cdot \frac{\csc 2x + \cot 2x}{\csc 2x + \cot 2x} 2dx \\ &= \int \frac{\csc^2 2x + \csc 2x \cot 2x}{\csc 2x + \csc 2x \cot 2x} 2dx = -\ln|\csc 2x + \cot 2x| + c \\ &= \ln \left| \frac{1}{\csc 2x + \cot 2x} \right| + c \end{aligned}$$

$$2) \int_0^1 \frac{dx}{2x+1}$$

$$= \frac{1}{2} \int_0^1 \frac{2dx}{2x+1} = \frac{1}{2} \left| \ln|2x+1| \right|_0^1 = \frac{1}{2} |\ln 3 - \ln 1| = \frac{1}{2} \ln 3 = \ln \sqrt{3}$$

$$3) \int_0^{\frac{\pi}{3}} \sec x \tan x dx$$

$$= [\sec x]_0^{\frac{\pi}{3}} = \sec \frac{\pi}{3} - \sec 0 = 2 - 1 = 1$$

$$4) \int_1^{10} \frac{\ln|x+z|}{x+z} dx$$

$$\begin{aligned} &= \int_1^{10} \ln|x+z| \cdot \frac{1}{x+z} dx = \frac{1}{2} [\ln^2|x+z|]_1^{10} \\ &= \frac{1}{2} [\ln^2|10+z| - \ln^2|1+z|] \end{aligned}$$

$$5) \int \frac{\cos^4 x - \sin^4 x}{\sqrt{1 + \cos 4x}} dx$$

$$\begin{aligned} &= \int \frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{\sqrt{1 + 2\cos^2 2x - 1}} dx = \int \frac{\cos^2 x - \sin^2 x}{\sqrt{2\cos^2 2x}} dx \\ &= \pm \frac{1}{\sqrt{2}} \int \frac{\cos 2x}{\cos 2x} dx = \pm \frac{1}{\sqrt{2}} \int dx = \pm \frac{1}{\sqrt{2}} x + c \end{aligned}$$

$$6) \int_2^5 \frac{dx}{5x-1}$$

$$\begin{aligned} &= \frac{1}{5} \int_2^5 \frac{5dx}{5x-1} = \frac{1}{5} [\ln|5x-1|]_2^5 = \frac{1}{5} [\ln 24 - \ln 9] \\ &= \frac{1}{5} [\ln 8 * 3 - \ln 3^2] = \frac{1}{5} [\ln 8 + \ln 3 - 2\ln 3] = \frac{1}{5} [\ln 8 - \ln 3] \\ &= \frac{1}{5} [\ln \frac{8}{3}] = \ln \sqrt[5]{\frac{8}{3}} \end{aligned}$$

$$7) \int \frac{\tan^2 \sqrt{x}}{\sqrt{x}} dx$$

$$\begin{aligned} &= \int \frac{\sec^2 \sqrt{x} - 1}{\sqrt{x}} dx = \int \left(\frac{\sec^2 \sqrt{x}}{\sqrt{x}} - \frac{1}{\sqrt{x}} \right) dx \\ &= \int \left(\sec^2 \sqrt{x} \cdot \frac{1}{\sqrt{x}} - x^{-\frac{1}{2}} \right) dx = 2\tan \sqrt{x} - 2\sqrt{x} + c \end{aligned}$$

$$8) \int \frac{\tan\sqrt{x}}{\sqrt{x}} dx$$

$$= \int \frac{\sin\sqrt{x}}{\cos\sqrt{x}} \cdot \frac{1}{\sqrt{x}} dx = -2 \int \frac{-\sin\sqrt{x}}{\cos\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} dx = -2 \ln|\cos\sqrt{x}| + c$$

$$9) \int \frac{2\csc^2 3x}{\sec 3x \sqrt{\sin 3x}} dx$$

$$= \int \frac{2 \cdot \frac{1}{\sin^2 3x}}{\frac{1}{\cos 3x} \sin^2 3x} dx = 2 \int \frac{1}{\sin^2 3x} \cdot \frac{1}{\frac{1}{\cos 3x}} \cdot \cos 3x dx$$

$$= 2 \int \frac{\cos 3x}{\sin^2 3x} dx = 2 \int \sin^{-2} 3x \cos 3x dx$$

$$= \frac{2}{3} \cdot \frac{\sin^{-3} 3x}{\frac{-3}{2}} + c = -\frac{4}{9} \sin^{-3} 3x + c$$

$$= -\frac{4}{9} \cdot \frac{1}{\sin^2 3x} + c = -\frac{4}{9} \cdot \frac{1}{\sqrt{\sin^3 3x}} + c$$

$$10) \int \frac{1+\ln x}{x} dx$$

$$= \int \left(\frac{1}{x} + \frac{\ln x}{x} \right) dx = \int \left(\frac{1}{x} + \ln x \cdot \frac{1}{x} \right) dx = \ln|x| + \frac{1}{2} \ln^2 x + c$$

$$11) \int \frac{dx}{x\sqrt{\ln x}}$$

$$= \int \ln^{-\frac{1}{2}} x \cdot \frac{1}{x} dx = 2 \ln^{\frac{1}{2}} x + c = 2\sqrt{\ln x} + c$$

$$12) \int \frac{e^{-\tan x}}{\cos^2 x} dx$$

$$= \int e^{-\tan x} \cdot \frac{1}{\cos^2 x} dx = \int e^{-\tan x} \sec^2 x dx = -e^{-\tan x} + c$$

$$13) \int \frac{(\sin x - 1)(\cos x + 2)}{\cos x} dx$$

$$= \int \frac{(\sin x \cos x + 2 \sin x - \cos x - 2)}{\cos x} dx = \int \left(\frac{\sin x \cos x}{\cos x} + 2 \frac{\sin x}{\cos x} - \frac{\cos x}{\cos x} - \frac{2}{\cos x} \right) dx$$

$$= \int \left(\sin x + 2 \frac{\sin x}{\cos x} - 1 - 2 \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} \right) dx$$

$$= \int \left(\sin x + 2 \frac{\sin x}{\cos x} - 1 - 2 \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \right) dx$$

$$= -\cos x - 2 \ln |\cos x| - x - 2 \ln |\sec x + \tan x| + c$$

$$14) \int_0^4 \frac{dx}{(1+\sqrt{x})^4 \sqrt{x}}$$

$$= \int_0^4 (1 + \sqrt{x})^{-4} \frac{1}{\sqrt{x}} dx = \left[2 \cdot \frac{(1 + \sqrt{x})^{-3}}{-3} \right]_0^4$$

$$= \left[\frac{-2}{3(1 + \sqrt{x})^3} \right]_0^4 = \left[\frac{-2}{3(2+1)^3} + \frac{2}{3(1)} \right] = \left[-\frac{2}{81} + \frac{2}{3} \right] = \frac{-2+54}{81} = \frac{52}{81}$$

$$15) \int \frac{\cos^4 x}{\sin^2 x} dx$$

$$\begin{aligned} &= \int \frac{\cos^2 x \cos^2 x}{\sin^2 x} dx = \int \frac{\cos^2 x (1 - \sin^2 x)}{\sin^2 x} dx = \int \frac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x} dx \\ &= \int \left(\frac{\cos^2 x}{\sin^2 x} - \frac{\cos^2 x \sin^2 x}{\sin^2 x} \right) dx = \int \left(\frac{\cos^2 x}{\sin^2 x} - \cos^2 x \right) dx \\ &= \int (\cot^2 x - \cos^2 x) dx = \int \left([\csc^2 x - 1] - \frac{1}{2}(1 + \cos 2x) \right) dx \\ &= \int \left(\csc^2 x - \frac{3}{2} - \frac{1}{2} \cos 2x \right) dx \\ &= -\cot x - \frac{3}{2}x - \frac{1}{4} \sin 2x + C \end{aligned}$$

$$16) \int \frac{\cos^3 x}{\sin^2 x} dx$$

$$\begin{aligned} &= \int \frac{\cos x (1 - \sin^2 x)}{\sin^2 x} dx = \int \frac{\cos x - \cos x \sin^2 x}{\sin^2 x} dx \\ &= \int \left(\frac{\cos x}{\sin^2 x} - \frac{\cos x \sin^2 x}{\sin^2 x} \right) dx = \int (\csc x \cot x - \cos x) dx \\ &= -\csc x - \sin x + C \end{aligned}$$

$$17) \int \frac{-\sin^2 x}{\cos x} dx$$

$$\begin{aligned}&= \int \frac{-1 + \cos^2 x}{\cos x} dx = \int \left(\frac{-1}{\cos x} + \frac{\cos^2 x}{\cos x} \right) dx \\&= \int \left(-\sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} + \cos x \right) dx = \int \left(-\frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} + \cos x \right) dx \\&= -\ln |\sec x + \tan x| + \sin x + c\end{aligned}$$

$$18) \int \frac{\tan^3 x}{\sqrt{\sec x}} dx$$

$$\begin{aligned}&= \int \sec^{-\frac{1}{2}} x \tan x \cdot \tan^2 x dx = \int \sec^{-\frac{1}{2}} x \tan x (\sec^2 x - 1) dx \\&= \int \left(\sec^{\frac{3}{2}} x \tan x - \sec^{-\frac{1}{2}} x \tan x \right) dx \\&= \int \left(\sec^{\frac{1}{2}} x \sec x \tan x - \sec^{-\frac{3}{2}} x \sec x \tan x \right) dx \\&= \frac{2}{3} \sec^{\frac{3}{2}} x + 2 \sec^{-\frac{1}{2}} x + c \\&= \frac{2}{3} \sqrt{\sec^3 x} + \frac{2}{\sqrt{\sec x}} + c\end{aligned}$$

$$19) \int \tan^3 3y \sec^3 3y dy$$

$$\begin{aligned} &= \int \tan 3y (\sec^2 3y - 1) \sec^3 3y dy \\ &= \int (\sec^4 3y \sec 3y \tan 3y - \sec^2 3y \sec 3y \tan 3y) dy \\ &= \frac{1}{3} \cdot \frac{\sec^5 3y}{5} - \frac{1}{3} \cdot \frac{\sec^3 3y}{3} + c = \frac{1}{15} \sec^5 3y - \frac{1}{9} \sec^3 3y + c \end{aligned}$$

$$20) \int \sqrt{e^{2x} + e^{-2x} + 2} dx$$

$$\begin{aligned} &= \int \sqrt{(e^x)^2 + 2e^0 + (e^{-x})^2} dx = \int \sqrt{(e^x + e^{-x})^2} dx \\ &= \pm \int (e^x + e^{-x}) dx = \pm (e^x - e^{-x}) + c \end{aligned}$$

$$21) \int (1 + e^x) e^x dx$$

$$= \int (e^x + e^{2x}) dx = e^x + \frac{1}{2} e^{2x} + c$$

$$22) \int e^{x^2-x} (2x-1) dx$$

$$= e^{x^2-x} + c$$

$$23) \int \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx$$

$$= \int e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} dx = e^{\sqrt{x}} + c$$

$$24) \int \frac{8dx}{2x+3}$$

$$= 4 \int \frac{2dx}{2x+3} = 4 \ln|2x+3| + c = \ln|(2x+3)^4| + c$$

$$25) \int \frac{x dx}{4x^2+1}$$

$$= \frac{1}{8} \int \frac{8x}{4x^2+1} dx = \frac{1}{8} \ln|4x^2+1| + c = \ln|\sqrt[8]{4x^2+1}| + c$$

$$26) \int \sin 2x (\sin x - 4)^4 (\sin x + 4)^4 dx$$

$$= \int \sin 2x [(\sin x - 4)(\sin x + 4)]^4 dx$$

$$= \int \sin 2x (\sin^2 x - 16)^4 dx = \int (\sin^2 x - 16)^4 2 \sin x \cos x dx$$

$$= \frac{1}{5} (\sin^2 x - 16)^5 + c$$

$$27) \int \sin x (1 - \sin x)^3 (1 + \sin x)^3 dx$$

$$= \int \sin x [(1 - \sin x)(1 + \sin x)]^3 dx$$

$$= \int \sin x (1 - \sin^2 x)^3 dx = \int \sin x (\cos^2 x)^3 dx$$

$$= \int \cos^6 x \sin x dx = -\frac{1}{7} \cos^7 x + c$$

$$28) \int \frac{\cos^3 x - 4}{\cos x} dx$$

$$\begin{aligned} &= \int \left(\frac{\cos^3 x}{\cos x} - \frac{4}{\cos x} \right) dx = \int (\cos^2 x - 4 \sec x) dx \\ &= \int \left[\left(\frac{1}{2} + \frac{1}{2} \cos 2x \right) - 4 \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} \right] dx \\ &= \frac{1}{2} x + \frac{1}{4} \sin 2x - 4 \ln |\sec x + \tan x| + c \end{aligned}$$

$$29) \int \frac{dx}{\csc 2x - \cot 2x}$$

$$\begin{aligned} &= \int \frac{1}{\frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x}} dx = \int \frac{\sin 2x}{1 - \cos 2x} dx \\ &= \frac{1}{2} \int \frac{2 \sin 2x}{1 - \cos 2x} dx = \frac{1}{2} \ln |1 - \cos 2x| + c \\ &= \ln |\sqrt{1 - \cos 2x}| + c \end{aligned}$$

$$30) \int \frac{x-1}{\sqrt[3]{x^2} + \sqrt[3]{x+1}} dx$$

$$\begin{aligned} &= \int \frac{(\sqrt[3]{x-1})(\sqrt[3]{x^2} + \sqrt[3]{x+1})}{\sqrt[3]{x^2} + \sqrt[3]{x+1}} dx = \int (\sqrt[3]{x} - 1) dx \\ &= \int \left(x^{\frac{1}{3}} - 1 \right) dx = \frac{3}{4} x^{\frac{4}{3}} - x + c \\ &= \frac{3}{4} \sqrt[3]{x^4} - x + c \end{aligned}$$

$$31) \int \frac{1}{1+\cos 3x} dx$$

$$\begin{aligned} &= \int \frac{1}{1+\cos 3x} \cdot \frac{1-\cos 3x}{1-\cos 3x} dx = \int \frac{1-\cos 3x}{1-\cos^2 3x} dx = \int \frac{1-\cos 3x}{\sin^2 3x} dx \\ &= \int \left(\frac{1}{\sin^2 3x} - \frac{\cos 3x}{\sin^2 3x} \right) dx = \int (\csc^2 3x - \csc 3x \cot 3x) dx \\ &= -\frac{1}{3} \cot 3x + \frac{1}{3} \csc 3x + c \end{aligned}$$

$$32) \int \frac{dx}{\sqrt{x-x}}$$

$$\begin{aligned} &= \int \frac{dx}{\sqrt{x}(1-\sqrt{x})} = -2 \int \frac{-1}{2\sqrt{x}} \cdot \frac{1}{(1-\sqrt{x})} dx = -2 \ln|1-\sqrt{x}| + c \\ &= \ln \left| \frac{1}{(1-\sqrt{x})^2} \right| + c \end{aligned}$$

$$33) \int (\cos x - \sin 2x)^2 dx$$

$$\begin{aligned} &= \int (\cos x - 2\sin x \cos x)^2 dx = \int \cos^2 x (1 - 2\sin x)^2 dx \\ &= \int \cos^2 x (1 - 4\sin x + 4\sin^2 x) dx \\ &= \int (\cos^2 x - 4\cos^2 x \sin x + 4\sin^2 x \cos^2 x) dx \\ &= \int \left[\frac{1}{2} + \frac{1}{2} \cos 2x - 4\cos^2 x \sin x + (2\sin x \cos x)^2 \right] dx \\ &= \int \left[\frac{1}{2} + \frac{1}{2} \cos 2x - 4\cos^2 x \sin x + \sin^2 2x \right] dx \\ &= \int \left[\frac{1}{2} + \frac{1}{2} \cos 2x - 4\cos^2 x \sin x + \frac{1}{2} - \frac{1}{2} \cos 4x \right] dx \\ &= x + \frac{1}{4} \sin 2x + \frac{4}{3} \cos^3 x - \frac{1}{8} \sin 4x + c \end{aligned}$$

حل مجموعة من التكاملات من كرويات الرياضيات حسين فاضل الخماسي

$$34) \int \sqrt{x} e^{2x\sqrt{x}} dx$$

$$\begin{aligned} &= \int \sqrt{x} (e^{2x\sqrt{x}})^{\frac{1}{2}} dx = \int \sqrt{x} e^{x\sqrt{x}} dx = \int \sqrt{x} e^{x^{\frac{3}{2}}} dx \\ &= \int e^{x^{\frac{3}{2}}} x^{\frac{1}{2}} dx = \frac{2}{3} e^{x^{\frac{3}{2}}} + c = \frac{2}{3} e^{\sqrt{x^3}} + c \end{aligned}$$

$$35) \int \sec^{\frac{1}{2}} x \tan x dx$$

$$= \int \sec^{-\frac{1}{2}} x \sec x \tan x dx = 2 \sec^{\frac{1}{2}} x + c = 2\sqrt{\sec x} + c$$

$$36) \int \frac{e^x}{\sqrt{1-e^x}} dx$$

$$= \int (1 - e^x)^{-\frac{1}{2}} e^x dx = -2 (1 - e^x)^{\frac{1}{2}} + c = -2\sqrt{1 - e^x} + c$$

$$37) \int \frac{\tan 2x}{\sqrt{\cos 2x}} dx$$

$$\begin{aligned} &= \int \frac{1}{\sqrt{\cos 2x}} \tan 2x dx = \int \sqrt{\sec 2x} \tan 2x dx \\ &= \int \sec^{-\frac{1}{2}} 2x \sec 2x \tan 2x dx = \frac{1}{2} \cdot 2 \sec^{\frac{1}{2}} 2x + c = \sqrt{\sec 2x} + c \end{aligned}$$

OR:



حل مجموعه من التكاملات من كروبات الرياضيات حسين فاضل الخامس

$$\begin{aligned}
 38) \int \frac{\sin^4 x}{\sin x + \frac{1}{2} \sin 2x} dx \\
 = \int \frac{\sin^4 x}{\sin x + \sin x \cos x} dx = \int \frac{\sin^4 x}{\sin x(1+\cos x)} dx = \int \frac{\sin^3 x}{1+\cos x} dx \\
 = \int \frac{\sin^2 x \sin x}{1+\cos x} dx = \int \frac{(1-\cos^2 x)\sin x}{1+\cos x} dx \\
 = \int \frac{(1+\cos x)(1-\cos x)\sin x}{1+\cos x} dx = \int (1 - \cos x) \sin x dx \\
 = \int (\sin x - \sin x \cos x) dx = -\cos x - \frac{1}{2} \sin^2 x + c
 \end{aligned}$$

$$\begin{aligned}
 39) \int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \sin^2 x (1 + \cot^2 x) dx \\
 = \int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \sin^2 x \cdot \csc^2 x dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin^2 x}{\sin^2 x} dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} dx \\
 = [x]_{-\frac{\pi}{4}}^{\frac{\pi}{3}} = \frac{\pi}{3} + \frac{\pi}{4} = \frac{7\pi}{12}
 \end{aligned}$$

$$\begin{aligned}
 40) \int \cos^3 x dx \\
 = \int \cos^2 x \cos x dx = \int (1 - \sin^2 x) \cos x dx \\
 = \int (\cos x - \sin^2 x \cos x) dx = \sin x - \frac{1}{3} \sin^3 x + c
 \end{aligned}$$

$$41) \int \tan^3 x \sec^4 x \, dx$$

$$\begin{aligned} &= \int \tan^3 x \sec^2 x \cdot \sec^2 x \, dx = \int \tan^3 x \sec^2 x (\tan^2 x + 1) \, dx \\ &= \int (\tan^5 x \sec^2 x + \tan^3 x \sec^2 x) \, dx \\ &= \frac{1}{6} \tan^6 x + \frac{1}{4} \tan^4 x + c \end{aligned}$$

$$42) \int \tan^3 x \sec^3 x \, dx$$

$$\begin{aligned} &= \int \tan^2 x \tan x \sec^3 x \, dx = \int \sec^3 x \tan x (\sec^2 x - 1) \, dx \\ &= \int (\sec^4 x \sec x \tan x - \sec^2 x \sec x \tan x) \, dx \\ &= \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + c \end{aligned}$$

$$43) \int x^7 \tan(8x^8 + 6) \, dx$$

$$= -\frac{1}{64} \int \frac{\sin(8x^8 + 6)}{\cos(8x^8 + 6)} (-64x^7) \, dx = -\frac{1}{64} \ln |\cos(8x^8 + 6)| + c$$

$$44) \int \frac{e^x}{x^2} \, dx$$

$$= \int e^x \cdot \frac{1}{x^2} \, dx = -e^x \cdot \frac{1}{x} + c$$

$$45) \int \frac{\cos(3+5\ln 9x)}{7x} dx$$

$$= \frac{1}{7} \int \cos(3 + 5\ln 9x) \cdot \frac{1}{x} dx = \frac{1}{35} \sin(3 + 5\ln 9x) + c$$

$$46) \int \frac{\sqrt[3]{\ln x - 5}}{x} dx$$

$$= \int (\ln x - 5)^{\frac{1}{3}} \cdot \frac{1}{x} dx = \frac{3}{4} (\ln x - 5)^{\frac{4}{3}} + c = \frac{3}{4} \sqrt[3]{(\ln x - 5)^4} + c$$

$$47) \int \frac{x^5 + 6x^3 + 9x}{(x^2 + 3)^3} dx$$

$$= \int \frac{x(x^4 + 6x^2 + 9)}{(x^2 + 3)^3} dx = \int \frac{x(x^2 + 3)^2}{(x^2 + 3)^3} dx = \int \frac{x}{x^2 + 3} dx$$

$$= \frac{1}{2} \ln|x^2 + 3| + c = \ln|\sqrt{x^2 + 3}| + c$$

$$48) \int_1^e \frac{3 - \ln x}{x} dx$$

$$= \int_1^e (3 - \ln x) \cdot \frac{1}{x} dx = -\frac{1}{2} [(3 - \ln x)^2]_1^e$$

$$= -\frac{1}{2} [(3 - \ln e)^2 - (3 - \ln 1)^2] = -\frac{1}{2} [4 - 9]$$

$$= -\frac{1}{2} (-5) = \frac{5}{2}$$

$$49) \int \cot^3 x \csc^5 x dx$$

$$\begin{aligned} &= \int (\csc^2 x - 1) \cot x \csc^5 x dx = \int (\csc^7 x \cot x - \csc^5 x \cot x) dx \\ &= \int (\csc^6 x \csc x \cot x - \csc^4 x \csc x \cot x) dx \\ &= -\frac{1}{7} \csc^7 x + \frac{1}{5} \csc^5 x + c \end{aligned}$$

$$50) \int \frac{\sqrt[3]{\sin x}}{\cos^3 x} dx$$

$$\begin{aligned} &= \int \frac{\sin^3 x}{\cos^3 x} \cdot \frac{1}{\cos^2 x} dx = \int \left(\frac{\sin x}{\cos x} \right)^{\frac{1}{3}} \cdot \frac{1}{\cos^2 x} dx = \int \tan^{\frac{1}{3}} x \sec^2 x dx \\ &= \frac{3}{4} \tan^{\frac{4}{3}} x + c = \frac{3}{4} \sqrt[3]{\tan^4 x} + c \end{aligned}$$

$$51) \int x^{10} \left(\frac{2}{x} + \frac{3}{x^2} \right)^5 dx$$

$$\begin{aligned} &= \int (x^2)^5 \left(\frac{2}{x} + \frac{3}{x^2} \right)^5 dx = \int \left[x^2 \left(\frac{2}{x} + \frac{3}{x^2} \right) \right]^5 dx \\ &= \int (2x + 3)^5 dx = \frac{1}{12} (2x + 3)^6 + c \end{aligned}$$

$$52) \int \frac{dx}{\sin x}$$

$$= \int \csc x \cdot \frac{\csc x + \cot x}{\csc x + \cot x} dx = \int \frac{\csc^2 x + \csc x \cot x}{\csc x + \cot x} dx = -\ln |\csc x + \cot x| + c$$

$$53) \int \frac{x+5}{(x+2)^5} dx$$

$$\begin{aligned} &= \int \frac{x+2+3}{(x+2)^5} dx = \int \frac{x+2}{(x+2)^5} dx + 3 \int \frac{1}{(x+2)^5} dx \\ &= \int \frac{1}{(x+2)^4} dx + 3 \int \frac{1}{(x+2)^5} dx \\ &= \int (x+2)^{-4} dx + 3 \int (x+2)^{-5} dx \\ &= \frac{(x+2)^{-3}}{-3} + \frac{3(x+2)^{-4}}{-4} + c \\ &= \frac{-1}{3(x+2)^3} - \frac{3}{4(x+2)^4} + c \end{aligned}$$

$$54) \int x^3 \sqrt{\frac{6}{x^2} + \frac{5}{x^3}} dx$$

$$\begin{aligned} &= \int \sqrt[3]{x^3 \left(\frac{6}{x^2} + \frac{5}{x^3}\right)} dx = \int \sqrt[3]{6x + 5} dx \\ &= \int (6x + 5)^{\frac{1}{3}} dx = \frac{1}{6} \cdot \frac{3}{4} (6x + 5)^{\frac{4}{3}} + c \\ &= \frac{3}{8} \sqrt[3]{(6x + 5)^4} + c \end{aligned}$$

$$55) \int \frac{1}{\cos^2 x} dx$$

$$= \int \sec^2 x dx = \tan x + c$$

$$56) \int \frac{9x^2-1}{(3x+1)^4} dx$$

$$\begin{aligned} &= \int \frac{(3x+1)(3x-1)}{(3x+1)^4} dx = \int \frac{3x-1}{(3x+1)^3} dx \\ &= \int \frac{3x+1-2}{(3x+1)^3} dx = \int \frac{3x+1}{(3x+1)^3} dx - 2 \int \frac{1}{(3x+1)^3} dx \\ &= \int \frac{1}{(3x+1)^2} dx - 2 \int \frac{1}{(3x+1)^3} dx \\ &= \int (3x+1)^{-2} dx - 2 \int (3x+1)^{-3} dx \\ &= \frac{1}{3} \cdot \frac{(3x+1)^{-1}}{-1} - \frac{2}{3} \cdot \frac{(3x+1)^{-2}}{-2} + c \\ &= \frac{-1}{3(3x+1)} + \frac{1}{3(3x+1)^2} + c \end{aligned}$$

$$57) \int \sin^3 3x dx$$

$$\begin{aligned} &= \int \sin^2 3x \cdot \sin 3x dx = \int (1 - \cos^2 3x) \sin 3x dx \\ &= \int (\sin 3x - \cos^2 3x \sin 3x) dx \\ &= -\frac{1}{3} \cos 3x + \frac{1}{9} \cos^3 3x + c \end{aligned}$$

$$58) \int \frac{x^2-4}{x+2} dx$$

$$\begin{aligned} &= \int \frac{(x+2)(x-2)}{x+2} dx = \int (x-2) dx \\ &= \frac{x^2}{2} - 2x + c \end{aligned}$$

$$59) \int \sec^4 3x \, dx$$

$$\begin{aligned} &= \int \sec^2 3x \sec^2 3x \, dx = \int \sec^2 3x (\tan^2 3x + 1) \, dx \\ &= \int (\tan^2 3x \sec^2 3x + \sec^2 3x) \, dx \\ &= \frac{1}{3} \frac{\tan^3 3x}{3} + \frac{1}{3} \tan 3x + c = \frac{1}{9} \tan^3 3x + \frac{1}{3} \tan 3x + c \end{aligned}$$

$$60) \int \tan^3 2x \, dx$$

$$\begin{aligned} &= \int \tan^2 2x \tan 2x \, dx = \int (\sec^2 2x - 1) \tan 2x \, dx \\ &= \int (\tan 2x \sec^2 2x - \tan 2x) \, dx \\ &= \int \left(\tan 2x \sec^2 2x - \frac{\sin 2x}{\cos 2x} \right) \, dx \\ &= \frac{1}{4} \tan^2 2x + \frac{1}{2} \ln |\cos 2x| + c \end{aligned}$$

$$61) \int \frac{e^{\sqrt{x}} - 1}{\sqrt{x}} \, dx$$

$$\begin{aligned} &= \int \left(\frac{e^{\sqrt{x}}}{\sqrt{x}} - \frac{1}{\sqrt{x}} \right) \, dx = \int \left(e^{\sqrt{x}} \cdot \frac{1}{\sqrt{x}} - x^{-\frac{1}{2}} \right) \, dx \\ &= 2e^{\sqrt{x}} - 2x^{\frac{1}{2}} + c = 2e^{\sqrt{x}} - 2\sqrt{x} + c \end{aligned}$$

$$62) \int x \cos(x^2 - 4) \, dx$$

$$= \frac{1}{2} \sin(x^2 - 4) + c$$

$$63) \int \frac{x^2-4}{x} dx$$

$$= \int \left(\frac{x^2}{x} - \frac{4}{x} \right) dx = \int \left(x - \frac{4}{x} \right) dx = \frac{x^2}{2} - 4 \ln|x| + c$$

$$64) \int \frac{x}{(x^2-4)^4} dx$$

$$= \int (x^2 - 4)^{-4} x dx = \frac{1}{2} \cdot \frac{(x^2-4)^{-3}}{-3} + c$$

$$= \frac{-1}{6(x^2-4)^3} + c$$

$$65) \int \frac{x^4-7x^2+12}{x^2-4} dx$$

$$= \int \frac{(x^2-4)(x^2-3)}{x^2-4} dx = \int (x^2-3) dx = \frac{x^3}{3} - 3x + c$$

$$66) \int \frac{x^4-16}{x+2} dx$$

$$= \int \frac{(x^2-4)(x^2+4)}{x+2} dx = \int \frac{(x+2)(x-2)(x^2+4)}{x+2} dx$$

$$= \int (x-2)(x^2+4) dx = \int (x^3 - 2x^2 + 4x - 8) dx$$

$$= \frac{x^4}{4} - \frac{2}{3}x^3 + 2x^2 - 8x + c$$

$$\begin{aligned}67) \int \frac{x^3 - 4x}{\sqrt{x^2 - 4}} dx \\&= \int \frac{x(x^2 - 4)}{\sqrt{x^2 - 4}} dx = \int \frac{x(\sqrt{x^2 - 4})(\sqrt{x^2 - 4})}{\sqrt{x^2 - 4}} dx \\&= \int (x^2 - 4)^{\frac{1}{2}} x dx = \frac{1}{2} \cdot \frac{2}{3} (x^2 - 4)^{\frac{3}{2}} + c \\&= \frac{1}{3} \sqrt{(x^2 - 4)^3} + c\end{aligned}$$

$$\begin{aligned}68) \int e^{x^2 - 4} x dx \\&= \frac{1}{2} e^{x^2 - 4} + c\end{aligned}$$

$$\begin{aligned}69) \int e^{x^3 - 12x} (x^2 - 4) dx \\&= \frac{1}{3} e^{x^3 - 12x} + c\end{aligned}$$

$$\begin{aligned}70) \int \frac{e^{2x}}{e^{2x} - 4} dx \\&= \frac{1}{2} \ln |e^{2x} - 4| + c\end{aligned}$$

$$\begin{aligned}71) \int \frac{x}{x^2 - 4} dx \\&= \frac{1}{2} \ln |x^2 - 4| + c\end{aligned}$$

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$$72) \int \frac{x^2-4}{\frac{1}{3}x^3-4x} dx$$

$$= \ln \left| \frac{1}{3}x^3 - 4x \right| + c$$

$$73) \int \left(x^{\sqrt{2}} + \frac{2^{\sqrt{x}}}{\sqrt{x}} + \sqrt{2^x} - 2\sqrt{x} \right) dx$$

$$= \int \left(x^{\sqrt{2}} + 2^{\sqrt{x}} \cdot \frac{1}{\sqrt{x}} + 2^{\frac{1}{2}x} - 2x^{\frac{1}{2}} \right) dx$$

$$= \frac{x^{\sqrt{2}+1}}{\sqrt{2}+1} + \frac{2 \cdot 2^{\sqrt{x}}}{\ln 2} + 2 \cdot 2^{\frac{1}{2}x} \cdot \frac{1}{\ln 2} - \frac{4}{3} x^{\frac{3}{2}} + c$$

$$= \frac{x^{\sqrt{2}+1}}{\sqrt{2}+1} + \frac{2^{\sqrt{x}+1}}{\ln 2} + \frac{2\sqrt{2^x}}{\ln 2} - \frac{4}{3} \sqrt{x^3} + c$$

74)

$$\int \sqrt[3]{(\sin x - \cos x)(1 - \sin 2x)} dx$$

$$= \int \sqrt[3]{(\sin x - \cos x)(\sin^2 x + \cos^2 x - 2\sin x \cos x)} dx$$

$$= \int \sqrt[3]{(\sin x - \cos x)(\sin x - \cos x)^2} dx = \int \sqrt[3]{(\sin x - \cos x)^3} dx$$

$$= \int (\sin x - \cos x) dx = -\cos x - \sin x + c$$

OR

$$\begin{aligned} & \int \sqrt[3]{(\sin x - \cos x)(1 - \sin 2x)} dx \\ &= \int \sqrt[3]{(\sin x - \cos x)(\sin^2 x + \cos^2 x - 2\sin x \cos x)} dx \end{aligned}$$

$$\begin{aligned} & \int \sqrt[3]{-(\cos x - \sin x)(\cos x - \sin x)^2} dx = \\ & \int \sqrt[3]{-(\cos x - \sin x)^3} dx \\ &= - \int (\cos x - \sin x) dx = -[\sin x + \cos x] + c \\ &= -\sin x - \cos x + c \end{aligned}$$

75) $\int \sqrt[5]{\csc^{11} x} \cdot \sqrt[5]{\cos x} dx$

$$\begin{aligned} &= \int \sqrt[5]{\csc^{10} x} \cdot \sqrt[5]{\frac{\cos x}{\sin x}} dx = \int \sqrt[5]{\cot x} \cdot \sqrt[5]{(\csc^2)^5} dx \\ &= \int (\cot x)^{\frac{1}{5}} \csc^2 x dx = -\frac{\cot^{\frac{6}{5}} x}{\frac{6}{5}} + c \\ &= -\frac{5}{6} \sqrt[5]{\cot^6 x} + c \end{aligned}$$

76) $\int \frac{\ln^3 x + 1}{x \ln x - 2x} dx$

$$\begin{aligned} &= \int \frac{\ln^3 x - 8 + 9}{x \ln x - 2x} dx = \int \frac{(\ln x - 2)(\ln^2 x + 2\ln x + 4)}{x(\ln x - 2)} dx + \int \frac{9}{x(\ln x - 2)} dx \\ &= \int \left(\ln^2 x \cdot \frac{1}{x} + 2\ln x \cdot \frac{1}{x} + \frac{4}{x} \right) dx + 9 \int \frac{\frac{1}{x}}{\ln x - 2} dx \\ &= \frac{1}{3} \ln^3 x + \ln^2 x + 4\ln|x| + 9\ln|\ln x - 2| + c \end{aligned}$$

$$77) \int (x+2)(x+3)^{15} dx$$

$$\begin{aligned} &= \int (x+3-1)(x+3)^{15} dx = \int [(x+3)(x+3)^{15} - (x+3)^{15}] dx \\ &= \int [(x+3)^{16} - (x+3)^{15}] dx \\ &= \frac{1}{17}(x+3)^{17} - \frac{1}{16}(x+3)^{16} + c \end{aligned}$$

$$78) \int \frac{x^2+6x+8}{x^3+8x^2+16x} dx$$

$$\begin{aligned} &= \int \frac{(x+4)(x+2)}{x(x+4)^2} dx = \int \frac{x+2}{x^2+4x} dx \\ &= \frac{1}{2} \ln |x^2 + 4x| + c = \ln |\sqrt{x^2 + 4x} + c| \end{aligned}$$

$$79) \int \frac{\sin 2x}{\sqrt[3]{5+4\sin^2 x}} dx$$

$$\begin{aligned} &= \int \frac{\sin 2x}{(5+4\sin^2 x)^{\frac{1}{3}}} dx = \int (5+4\sin^2 x)^{-\frac{1}{3}} (2\sin x \cos x) dx \\ &= \frac{1}{4} \cdot \frac{3}{2} (5+\sin^2 x)^{\frac{2}{3}} + c = \frac{3}{8} \sqrt[3]{(5+4\sin^2 x)^2} + c \end{aligned}$$

$$80) \int 4^x (4^x + 1) dx$$

$$= \int (4^{2x} + 4^x) dx = \frac{4^{2x}}{2\ln 4} + \frac{4^x}{\ln 4} + c$$

$$81) \int_1^2 x^4 e^{-\ln x} dx$$

$$= \int_1^2 x^4 e^{\ln x - 1} dx = \int_1^2 x^4 x^{-1} dx = \int_1^2 x^3 dx$$

$$= \left[\frac{x^4}{4} \right]_1^2 = \left(\frac{16}{4} - \frac{1}{4} \right) = \frac{15}{4}$$

$$82) \int \frac{1}{\sqrt[3]{3x+5}} dx$$

$$= \int \frac{1}{(3x+5)^{\frac{1}{3}}} dx = \int (3x+5)^{-\frac{1}{3}} dx$$

$$= \frac{1}{3} \cdot \frac{3}{2} (3x+5)^{\frac{2}{3}} + c = \frac{1}{2} \sqrt[3]{(3x+5)^2} + c$$

$$83) \int \frac{-7x}{(x^2-10)^{\frac{7}{8}}} dx$$

$$= -7 \int (x^2 - 10)^{-\frac{7}{8}} x dx = -7 \cdot \frac{1}{2} \frac{(x^2-10)^{-7}}{-7} + c$$

$$= \frac{1}{2(x^2-10)^{\frac{7}{8}}} + c$$

$$84) \int \frac{\sin 8x}{(2-\cos 8x)^2} dx$$

$$= \int (2 - \cos 8x)^{-2} \sin 8x dx = \frac{1}{8} \frac{(2 - \cos 8x)^{-1}}{-1} + c$$

$$= \frac{-1}{8(2 - \cos 8x)} + c$$

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$$85) \int \sqrt[3]{\tan 2x + 5} \sec^2 2x \, dx$$

$$\begin{aligned} &= \int (\tan 2x + 5)^{\frac{1}{3}} \sec^2 2x \, dx = \frac{1}{2} \cdot \frac{3}{4} (\tan 2x + 5)^{\frac{4}{3}} + c \\ &= \frac{3}{8} \sqrt[3]{(\tan 2x + 5)^4} + c \end{aligned}$$

$$86) \int \sin 5x \cos 5x \, dx$$

$$= \frac{1}{5} \cdot \frac{1}{2} \sin^2 5x + c = \frac{1}{10} \sin^2 5x + c$$

$$\text{OR : } -\frac{1}{10} \cos^2 5x + c$$

$$87) \int \frac{\cos x}{(2+\sin x)^2} \, dx$$

$$\begin{aligned} &= \int (2 + \sin x)^{-2} \cos x \, dx = \frac{(2 + \sin x)^{-1}}{-1} + c \\ &= -\frac{1}{2 + \sin x} + c \end{aligned}$$

$$88) \int \frac{\sin 2x}{\cos^2 2x} \, dx$$

$$= \int \frac{\sin 2x}{\cos 2x} \cdot \frac{1}{\cos 2x} \, dx = \int \tan 2x \cdot \sec 2x \, dx = \frac{1}{2} \sec 2x + c$$

$$\text{OR: } \int \frac{\sin 2x}{\cos^2 2x} \, dx = \int (\cos 2x)^{-2} \sin 2x \, dx = -\frac{1}{2} \cdot \frac{(\cos 2x)^{-1}}{-1} + c$$

$$= \frac{1}{2} \cdot \frac{1}{\cos 2x} + c = \frac{1}{2} \sec 2x + c$$

$$89) \int \frac{\sqrt{x} - \frac{3}{\sqrt{x}}}{x^2} dx$$

$$\begin{aligned} &= \int \left(\frac{x^{\frac{1}{2}}}{x^2} - \frac{x^{-\frac{3}{2}}}{x^2} \right) dx = \int \left(x^{\frac{1}{2}}x^{-2} - x^{-\frac{5}{2}} \right) dx \\ &= \int \left(x^{-\frac{3}{2}} - x^{-\frac{5}{2}} \right) dx = -2x^{\frac{-1}{2}} + \frac{3}{2}x^{\frac{-2}{2}} + c \\ &= -\frac{2}{\sqrt{x}} + \frac{3}{2\sqrt{x^2}} + c \end{aligned}$$

$$90) \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (-\csc^2 x e^{\cot x}) dx$$

$$= [e^{\cot x}]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = e^{\cot \frac{\pi}{2}} - e^{\cot \frac{\pi}{4}} = e^0 - e^1 = 1 - e$$

$$91) \int_{\ln 2}^{\ln 4} \frac{dx}{e^{2x}}$$

$$\begin{aligned} &= \int_{\ln 2}^{\ln 4} e^{-2x} dx = -\frac{1}{2} [e^{-2x}]_{\ln 2}^{\ln 4} \\ &= -\frac{1}{2} [e^{-2\ln 4} - e^{-2\ln 2}] = -\frac{1}{2} [e^{\ln 4^{-2}} - e^{\ln 2^{-2}}] \\ &= -\frac{1}{2} [4^{-2} - 2^{-2}] = -\frac{1}{2} \left[\frac{1}{16} - \frac{1}{4} \right] = -\frac{1}{32} + \frac{1}{8} = \frac{3}{32} \end{aligned}$$

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$$\begin{aligned} 92) \int \frac{dx}{\sqrt[5]{x^2+16x+64}} \\ &= \int \frac{dx}{((x+8)^2)^{\frac{1}{5}}} = \int \frac{dx}{(x+8)^{\frac{2}{5}}} = \int (x+8)^{-\frac{2}{5}} dx \\ &= \frac{5}{3} (x+8)^{\frac{3}{5}} + c = \frac{5}{3} \sqrt[5]{(x+8)^3} + c \end{aligned}$$

$$\begin{aligned} 93) \int (\sec x \tan x - 1)^2 dx \\ &= \int (\tan^2 x \sec^2 x - 2 \sec x \tan x + 1) dx \\ &= \frac{1}{3} \tan^3 x - 2 \sec x + x + c \end{aligned}$$

$$\begin{aligned} 94) \int_0^1 \frac{x-1}{x-\sqrt{x}} dx \\ &= \int_0^1 \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)} dx = \int_0^1 (\sqrt{x}+1) \cdot \frac{1}{\sqrt{x}} dx \\ &= \left[(\sqrt{x}+1)^2 \right]_0^1 = \left((\sqrt{1}+1)^2 - (\sqrt{0}+1)^2 \right) = 4 - 1 = 3 \end{aligned}$$

$$\begin{aligned} 95) \int_{-1}^1 3^{3x-1} dx \\ &= \left[\frac{3^{3x-1}}{3 \ln 3} \right]_{-1}^1 = \frac{1}{3 \ln 3} [3^2 - 3^{-4}] = \frac{1}{3 \ln 3} \left[9 - \frac{1}{81} \right] \\ &= \frac{1}{3 \ln 3} \left(\frac{729-1}{81} \right) = \frac{728}{243 \ln 3} \end{aligned}$$

$$96) \int \sin 3x dx$$

$$= -\frac{1}{3} \cos 3x + c$$

$$97) \int \cos(3x - 1) dx$$

$$= \frac{1}{3} \sin(3x - 1) + c$$

$$98) \int \tan^3 x dx$$

$$\begin{aligned} &= \int \tan^2 x \tan x dx = \int (\sec^2 x - 1) \tan x dx \\ &= \int (\tan x \sec^2 x - \tan x) dx = \frac{1}{2} \tan^2 x + \ln |\cos x| + c \end{aligned}$$

$$99) \int \csc^4 x dx$$

$$\begin{aligned} &= \int \csc^2 x \csc^2 x dx = \int \csc^2 x (\cot^2 x + 1) dx \\ &= \int (\cot^2 x \csc^2 x + \csc^2 x) dx = -\frac{1}{3} \cot^3 x - \cot x + c \end{aligned}$$

$$100) \int \sin(\sin x) \cos x dx$$

$$= -\cos(\sin x) + c$$

$$101) \int \sqrt{\sec x} \sec x \tan x dx$$

$$= \int (\sec x)^{\frac{1}{2}} \sec x \tan x dx = \frac{2}{3} \sec^{\frac{3}{2}} x + c = \frac{2}{3} \sqrt{\sec^3 x} + c$$

$$102) \int \frac{dx}{1-\cos x}$$

$$\begin{aligned} &= \int \frac{1}{1-\cos x} \cdot \frac{1+\cos x}{1+\cos x} dx = \int \frac{1+\cos x}{1-\cos^2 x} dx = \int \frac{1+\cos x}{\sin^2 x} dx \\ &= \int (\csc^2 x + \csc x \cot x) dx = -\cot x - \csc x + c \end{aligned}$$

$$103) \int \frac{dx}{1+\sin x}$$

$$\begin{aligned} &= \int \frac{1}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x} dx = \int \frac{1-\sin x}{1-\sin^2 x} dx = \int \frac{1-\sin x}{\cos^2 x} dx \\ &= \int (\sec^2 x - \sec x \tan x) dx = \tan x - \sec x + c \end{aligned}$$

$$104) \int \cos^8 x \sin^3 x dx$$

$$\begin{aligned} &= \int \cos^8 x \sin^2 x \sin x dx = \int \cos^8 x \sin x (1 - \cos^2 x) dx \\ &= \int (\cos^8 x \sin x - \cos^{10} x \sin x) dx \\ &= -\frac{1}{9} \cos^9 x + \frac{1}{11} \cos^{11} x + c \end{aligned}$$

$$105) \int \sec x \csc^2 x dx$$

$$\begin{aligned} &= \int \sec x (\cot^2 x + 1) dx = \int (\sec x \cot^2 x + \sec x) dx \\ &= \int \left(\frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cos x} + \sec x \right) dx = \int \left(\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} + \sec x \right) dx \\ &= \int (\csc x \cot x + \sec x) dx = -\csc x + \ln|\sec x + \tan x| + c \end{aligned}$$

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$$106) \int \frac{\sin x \cos x}{3+2\cos^2 x} dx \\ = -\frac{1}{4} \ln |3 + 2\cos^2 x| + c$$

$$107) \int \frac{\cos x - \sin x}{(1+\sin 2x)^8} dx \\ = \int \frac{\cos x - \sin x}{(\sin^2 x + 2\sin x \cos x + \cos^2 x)^8} dx = \int \frac{\cos x - \sin x}{((\sin x + \cos x)^2)^8} dx \\ = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^{16}} dx = \int (\sin x + \cos x)^{-16} (\cos x - \sin x) dx \\ = -\frac{1}{15} (\sin x + \cos x)^{-15} + c = -\frac{1}{15(\sin x + \cos x)^{15}} + c$$

$$108) \int [\cot 2x \sqrt{\sin 2x} + \cos 2x] dx \\ = \int \left[\frac{\cos 2x}{\sin 2x} \cdot (\sin 2x)^{\frac{1}{2}} + \cos 2x \right] dx \\ = \int \left[(\sin 2x)^{-1} (\sin 2x)^{\frac{1}{2}} \cos 2x + \cos 2x \right] dx \\ = \int \left[(\sin 2x)^{\frac{-1}{2}} \cos 2x + \cos 2x \right] dx \\ = \frac{1}{2} \cdot 2(\sin 2x)^{\frac{1}{2}} + \frac{1}{2} \sin 2x + c \\ = \sqrt{\sin 2x} + \frac{1}{2} \sin 2x + c$$

$$109) \int_0^{\frac{3\pi}{4}} |\cos x| dx$$

$$= \cos x = 0 \rightarrow x = \frac{\pi}{2} \in \left[0, \frac{3\pi}{4}\right]$$

$$= \int_0^{\frac{\pi}{2}} \cos x dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \cos x dx = [\sin x]_0^{\frac{\pi}{2}} + [\sin x]_{\frac{\pi}{2}}^{\frac{3\pi}{4}}$$

$$= \left[\sin \frac{\pi}{2} - \sin 0 \right] + \left[\sin \frac{3\pi}{4} - \sin \frac{\pi}{2} \right] = (1 - 0) + \left(\frac{1}{\sqrt{2}} - 1 \right) = \frac{1}{\sqrt{2}}$$

$$110) \int \frac{1 - \sin 4x}{\sin 2x - \cos 2x} dx$$

$$= \int \frac{\sin^2 2x + \cos^2 2x - 2 \sin 2x \cos 2x}{\sin 2x - \cos 2x} dx$$

$$= \int \frac{\sin^2 2x - 2 \sin 2x \cos 2x + \cos^2 2x}{\sin 2x - \cos 2x} dx$$

$$= \int \frac{(\sin 2x - \cos 2x)^2}{\sin 2x - \cos 2x} dx = \int (\sin 2x - \cos 2x) dx$$

$$= -\frac{1}{2} \cos 2x - \frac{1}{2} \sin 2x + c$$

$$111) \int \frac{\sin^3 3x}{\cos 3x - 1} dx$$

$$= \int \frac{\sin^2 3x \sin 3x}{\cos 3x - 1} dx = \int \frac{(1 - \cos^2 3x) \sin 3x}{\cos 3x - 1} dx$$

$$= \int \frac{(1 - \cos 3x)(1 + \cos 3x) \sin 3x}{-(1 - \cos 3x)} dx = - \int (\sin 3x + \sin 3x \cos 3x) dx$$

$$= - \left[-\frac{1}{3} \cos 3x + \frac{1}{6} \sin^2 3x \right] + c = \frac{1}{3} \cos 3x - \frac{1}{6} \sin^2 3x + c$$

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$$112) \int \sqrt[5]{(1-3x)^2} dx$$

$$= \int ((1-3x)^2)^{\frac{1}{5}} dx = \int (1-3x)^{\frac{2}{5}} dx$$

$$= -\frac{1}{3} \cdot \frac{5}{7} (1-3x)^{\frac{7}{5}} + c = -\frac{5}{21} \sqrt[5]{(1-3x)^7} + c$$

$$113) \int \sin^4 x \cos^2 x dx$$

$$= \int \left[\left(\frac{1}{2} (1 - \cos 2x) \right)^2 \left(\frac{1}{2} (1 + \cos 2x) \right) \right] dx$$

$$= \frac{1}{8} \int \left[(1 - 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x) (1 + \cos 2x) \right] dx$$

$$= \frac{1}{8} \int \left[\left(\frac{3}{2} - 2\cos 2x + \frac{1}{2}\cos 4x \right) (1 + \cos 2x) \right] dx$$

$$= \frac{1}{8} \int \left[\frac{3}{2} - 2\cos 2x + \frac{1}{2}\cos 4x + \frac{3}{2}\cos 2x - 2\cos^2 2x + \frac{1}{2}\cos 2x \cos 4x \right] dx$$

$$= \frac{1}{8} \int \left[\frac{3}{2} - \frac{1}{2}\cos 2x + \frac{1}{2}\cos 4x - 1 - \cos 4x + \right.$$

$$\left. \frac{1}{2}(1 - 2\sin^2 2x)\cos 2x \right] dx$$

$$= \frac{1}{8} \int \left[\frac{1}{2} - \frac{1}{2}\cos 2x - \frac{1}{2}\cos 4x + \frac{1}{2}\cos 2x - \sin^2 2x \cos 2x \right] dx$$

$$= \frac{1}{8} \int \left[\frac{1}{2} - \frac{1}{2}\cos 4x - \sin^2 2x \cos 2x \right] dx$$

$$= \frac{1}{8} \left[\frac{1}{2}x - \frac{1}{8}\sin 4x - \frac{1}{6}\sin^3 2x \right] + c$$

$$= \frac{x}{16} - \frac{1}{64}\sin 4x - \frac{1}{48}\sin^3 2x + c$$

$$\begin{aligned}
 114) \int \frac{x^3 - 2x^2 + 1}{5x^5} dx \\
 &= \frac{1}{5} \int \left(\frac{x^3}{x^5} - \frac{2x^2}{x^5} + \frac{1}{x^5} \right) dx = \frac{1}{5} \int (x^{-2} - 2x^{-3} + x^{-5}) dx \\
 &= \frac{1}{5} \left[\frac{x^{-1}}{-1} - 2 \frac{x^{-2}}{-2} + \frac{x^{-4}}{-4} \right] + c = \frac{1}{5} \left[-\frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^4} \right] + c \\
 &= \frac{-1}{5x} + \frac{1}{5x^2} - \frac{1}{5x^4} + c
 \end{aligned}$$

$$\begin{aligned}
 115) \int \frac{\cos^2 x}{2\csc^2 x} dx + \int \frac{\sin^2 x}{2\sec^2 x} dx \\
 &= \int \frac{2\cos^2 x - 1}{2\csc^2 x} dx + \int \frac{2\sin x \cos x}{2\sec^2 x} dx \\
 &= \int \left(\frac{2\cos^2 x}{2\csc^2 x} - \frac{1}{2\csc^2 x} \right) dx + \int (\cos^3 x \sin x) dx \\
 &= \int \left[\left(\frac{1}{2} \sin^2 x \cos^2 x \right) - \left(\frac{1}{2} \sin^2 x \right) \right] dx + \int (\cos^3 x \sin x) dx \\
 &= \int \left[\frac{1}{4} \sin^2 2x - \frac{1}{2} \sin^2 x \right] dx + \int (\cos^3 x \sin x) dx \\
 &= \int \left[\frac{1}{4} \left(\frac{1}{2} - \frac{1}{2} \cos 4x \right) - \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \cos 2x \right) \right] dx + \int (\cos^3 x \sin x) dx \\
 &= \int \left[\frac{1}{8} - \frac{1}{8} \cos 4x - \frac{1}{4} + \frac{1}{4} \cos 2x \right] dx + \int (\cos^3 x \sin x) dx \\
 &= \int \left(-\frac{1}{4} - \frac{1}{8} \cos 4x + \frac{1}{4} \cos 2x \right) dx + \int (\cos^3 x \sin x) dx \\
 &= -\frac{1}{4}x - \frac{1}{32} \sin 4x + \frac{1}{8} \sin 2x - \frac{1}{4} \cos^4 x + c
 \end{aligned}$$

$$116) \int \left(\sqrt{x} - \frac{3}{\sqrt[3]{x}} - 1 \right) dx$$

$$= \int \left(x^{\frac{1}{2}} - 3x^{-\frac{1}{3}} - 1 \right) dx$$

$$= \frac{2}{3}x^{\frac{3}{2}} - 3 \cdot \frac{3}{2}x^{\frac{2}{3}} - x + c$$

$$= \frac{2}{3}\sqrt{x^3} - \frac{9}{2}\sqrt[3]{x^2} - x + c$$

$$117) \int \frac{\sqrt[3]{x^3+2}}{\sqrt[3]{x}} dx$$

$$= \int \left(\frac{x}{x^{\frac{1}{3}}} + \frac{2}{x^{\frac{1}{3}}} \right) dx = \int \left(x x^{-\frac{1}{3}} + 2 x^{-\frac{1}{3}} \right) dx$$

$$= \int \left(x^{\frac{2}{3}} + 2x^{-\frac{1}{3}} \right) dx = \frac{3}{5}x^{\frac{5}{3}} + 2 \cdot \frac{3}{2}x^{\frac{2}{3}} + c$$

$$= \frac{3}{5}\sqrt[3]{x^5} + 3\sqrt[3]{x^2} + c$$

$$118) \int \frac{1}{\cos^2 x \sin^2 x} dx$$

$$= \int \frac{\cos^2 x + \sin^2 x}{\cos^2 x \sin^2 x} dx = \int \left(\frac{\cos^2 x}{\cos^2 x \sin^2 x} + \frac{\sin^2 x}{\cos^2 x \sin^2 x} \right) dx$$

$$= \int \left(\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} \right) dx = \int (\csc^2 x + \sec^2 x) dx$$

$$= -\cot x + \tan x + c$$

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$$119) \int \frac{\cos^2 x}{\sin x + \cos x} dx$$

$$= \int \frac{\cos^2 x - \sin^2 x}{\sin x + \cos x} dx = \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{\sin x + \cos x} dx \\ = \int (\cos x - \sin x) dx = \sin x + \cos x + c$$

$$120) \int \cos^{-4} x dx$$

$$= \int \sec^4 x dx = \int \sec^2 x (\tan^2 x + 1) dx \\ = \int (\tan^2 x \sec^2 x + \sec^2 x) dx \\ = \frac{1}{3} \tan^3 x + \tan x + c$$

$$121) \int \frac{1 - \cos 2x}{\sin x (1 + \cos 2x)} dx$$

$$= \int \frac{1 - (1 - 2\sin^2 x)}{\sin x (1 + (2\cos^2 x - 1))} dx = \int \frac{2\sin^2 x}{\sin x (2\cos^2 x)} dx \\ = \int \frac{\sin x}{\cos^2 x} dx = \int \sec x \tan x dx = \sec x + c$$

$$122) \int \frac{12x - 24}{\sqrt[6]{x-2}} dx$$

$$= \int \frac{12(x-2)}{\sqrt[6]{x-2}} \cdot \frac{\sqrt[6]{(x-2)^5}}{\sqrt[6]{(x-2)^5}} dx = 12 \int \frac{(x-2)^{\frac{5}{6}}}{x-2} dx \\ = 12 \int (x-2)^{\frac{5}{6}} dx = 12 \cdot \frac{6}{11} (x-2)^{\frac{11}{6}} + c \\ = \frac{72}{11} \sqrt[6]{(x-2)^{11}} + c$$

$$OR: 12 \int \frac{x-2}{(x-2)^6} dx = 12 \int (x-2)(x-2)^{-6} dx \\ = 12 \int (x-2)^{\frac{5}{6}} dx = 12 \cdot \frac{6}{11} (x-2)^{\frac{11}{6}} + c \\ = \frac{72}{11} \sqrt[6]{(x-2)^{11}} + c$$

$$OR: 12 \int \frac{x-2}{(x-2)^6} dx = 12 \int \frac{(x-2)^{\frac{1}{6}}(x-2)^{\frac{5}{6}}}{(x-2)^6} dx \\ = 12 \int (x-2)^{\frac{5}{6}} dx = 12 \cdot \frac{6}{11} (x-2)^{\frac{11}{6}} + c \\ = \frac{72}{11} \sqrt[6]{(x-2)^{11}} + c$$

123) $\int \frac{\cos 5x}{e^{\sin 5x}} dx$

$$= \int (e^{\sin 5x})^{-1} \cos 5x dx = \int e^{-\sin 5x} \cos 5x dx \\ = -\frac{1}{5} e^{-\sin 5x} + c = \frac{-1}{5e^{\sin 5x}} + c$$

124) $\int \frac{4 \csc^2 2x}{\sqrt{1-3 \cot 2x}} dx$

$$= -\frac{4}{6} \int (1-3 \cot 2x)^{\frac{-1}{2}} (-6 \csc^2 2x) dx \\ = -\frac{2}{3} \cdot 2 (1-3 \cot 2x)^{\frac{1}{2}} + c = -\frac{4}{3} \sqrt{1-3 \cot 2x} + c$$

$$\begin{aligned}
 125) \int x^{10} \left(5 - \frac{4}{x}\right)^{10} dx \\
 &= \int \left[x\left(5 - \frac{4}{x}\right)\right]^{10} dx = \int (5x - 4)^{10} dx \\
 &= \frac{1}{5} \cdot \frac{1}{11} (5x - 4)^{11} + c = \frac{1}{55} (5x - 4)^{11} + c
 \end{aligned}$$

$$\begin{aligned}
 126) \int \frac{e^x \operatorname{sece}^x \tan e^x}{\operatorname{sece}^x + e} dx \\
 &= \ln|\operatorname{sece}^x + e| + c
 \end{aligned}$$

$$\begin{aligned}
 127) \int dz &= \int zx(a - x^2)^{-1} dx + \int (a - x^2)^{\frac{-1}{2}} dy \\
 &= \int dz = \int \frac{zx}{(a - x^2)} dx + \int \frac{dy}{\sqrt{a - x^2}} \\
 &= z = -\frac{z}{2} \ln|a - x^2| + \frac{y}{\sqrt{a - x^2}} + c
 \end{aligned}$$

$$\begin{aligned}
 128) \int \frac{1}{x^2 + 3x + 2} dx \\
 &= \int \frac{x+2-x-1}{x^2 + 3x + 2} dx = \int \frac{x+2}{x^2 + 3x + 2} dx - \int \frac{x+1}{x^2 + 3x + 2} dx \\
 &= \int \frac{x+2}{(x+2)(x+1)} dx - \int \frac{x+1}{(x+2)(x+1)} dx \\
 &= \int \frac{1}{x+1} dx - \int \frac{1}{x+2} dx \\
 &= \ln|x+1| - \ln|x+2| + c
 \end{aligned}$$

$$129) \int \frac{x+5}{(x+2)^7} dx$$

$$\begin{aligned} &= \int \frac{x+2+5}{(x+2)^7} dx = \int \frac{x+2}{(x+2)^7} dx + 3 \int \frac{1}{(x+2)^7} dx \\ &= \int \frac{1}{(x+2)^6} dx + 3 \int \frac{1}{(x+2)^7} dx = \int (x+2)^{-6} dx + 3 \int (x+2)^{-7} dx \\ &= \frac{(x+2)^{-5}}{-5} + \frac{3(x+2)^{-6}}{-6} + c = \frac{-1}{5(x+2)^5} - \frac{1}{2(x+2)^6} + c \end{aligned}$$

$$130) \int_0^8 \frac{x}{x+1} dx$$

$$\begin{aligned} &= \int_0^8 \frac{x+1-1}{x+1} dx = \int_0^8 \frac{x+1}{x+1} dx - \int_0^8 \frac{1}{x+1} dx \\ &= \int_0^8 dx - \int_0^8 \frac{1}{x+1} dx = [x]_0^8 - [\ln|x+1|]_0^8 \\ &= (8 - 0) - (\ln 9 - \ln 1) = 8 - \ln 9 \end{aligned}$$

$$131) \int \frac{2\sin^3 \sqrt[3]{x}}{\sqrt[3]{x^2}} dx$$

$$= 2 \int \sin x^{\frac{1}{3}} \cdot x^{\frac{-2}{3}} dx = -6 \cos^3 \sqrt[3]{x} + c$$

$$132) \int (\cos x - \sin x)^4 \cos 2x dx$$

$$\begin{aligned} &= \int (\cos^2 x - 2\sin x \cos x + \sin^2 x)^2 \cos 2x dx \\ &= \int (1 - 2\sin 2x + \sin^2 x)^2 \cos 2x dx \\ &= \int (\cos 2x - 2\sin 2x \cos 2x + \sin^2 2x \cos 2x) dx \\ &= \frac{1}{2} \sin 2x - \frac{1}{2} \sin^2 2x + \frac{1}{6} \sin^3 2x + c \end{aligned}$$

$$OR: \int (\cos x - \sin x)^4 \cos 2x \, dx$$

$$= \int (\cos^2 x - 2\sin x \cos x + \sin^2 x)^2 \cos 2x \, dx$$

$$= \int (1 - \sin 2x)^2 \cos 2x \, dx = \int (1 - 2\sin 2x + \sin^2 x) \cos 2x \, dx$$

$$= \int (\cos 2x - \sin 4x + \sin^2 2x \cos 2x) \, dx$$

$$= \frac{1}{2} \sin 2x + \frac{1}{4} \cos 4x + \frac{1}{6} \sin^3 2x + c$$

$$OR: \int (\cos x - \sin x)^4 \cos 2x \, dx$$

$$= \int (\cos^2 x - 2\sin x \cos x + \sin^2 x)^2 \cos 2x \, dx$$

$$= \int (1 - \sin 2x)^2 \cos 2x \, dx = -\frac{1}{2} \cdot \frac{1}{3} (1 - \sin 2x)^3 + c$$

$$= -\frac{1}{6} (1 - \sin 2x)^3 + c$$

$$133) \int \sin 4x \csc x \, dx$$

$$= \int 2\sin 2x \cos 2x \csc x \, dx = \int 4\sin x \cos x \cos 2x \cdot \frac{1}{\sin x} \, dx$$

$$= 4 \int \cos x \cos 2x \, dx = 4 \int \cos x (1 - 2\sin^2 x) \, dx$$

$$= 4 \int (\cos x - 2\sin^2 x \cos x) \, dx = 4 \left[\sin x - \frac{2}{3} \sin^3 x \right] + c$$

$$= 4\sin x - \frac{8}{3} \sin^3 x + c$$

$$134) \int \sec^2 x \csc^2 x \, dx$$

$$\begin{aligned} &= \int (\tan^2 x + 1) \csc^2 x \, dx = \int (\tan^2 x \csc^2 x + \csc^2 x) \, dx \\ &= \int \left(\frac{\sin^2 x}{\cos^2 x \sin^2 x} + \csc^2 x \right) \, dx = \int (\sec^2 x + \csc^2 x) \, dx \\ &= \tan x - \cot x + c \end{aligned}$$

$$\begin{aligned} \text{OR: } &\int \sec^2 x \csc^2 x \, dx = \int \frac{1}{\sin^2 x \cos^2 x} \, dx \\ &= \int \frac{4}{4 \sin^2 x \cos^2 x} \, dx = 4 \int \frac{1}{(2 \sin x \cos x)^2} \, dx \\ &= 4 \int \frac{1}{\sin^2 2x} \, dx = 4 \int \csc^2 2x \, dx = -2 \cot 2x + c \end{aligned}$$

$$135) \int \frac{1-\cos 2x}{1+\cos 2x} \, dx$$

$$\begin{aligned} &= \int \frac{1-(1-2\sin^2 x)}{1+2\cos^2 x-1} \, dx = \int \frac{2\sin^2 x}{2\cos^2 x} \, dx = \int \tan^2 x \, dx \\ &= \int (\sec^2 x - 1) \, dx = \tan x - x + c \\ \text{OR: } &\int \frac{1-\cos 2x}{1+\cos 2x} \, dx = \int \frac{1-\cos 2x}{1+\cos 2x} \cdot \frac{1-\cos 2x}{1-\cos 2x} \, dx \\ &= \int \frac{1-2\cos 2x+\cos^2 2x}{1-\cos^2 2x} \, dx = \int \left(\frac{1}{\sin^2 2x} - \frac{2\cos 2x}{\sin^2 2x} + \frac{\cos^2 2x}{\sin^2 2x} \right) \, dx \\ &= \int (\csc^2 2x - 2\csc 2x \cot 2x + \cot^2 2x) \, dx \\ &= \int (\csc^2 2x - 2\csc 2x \cot 2x + \csc^2 2x - 1) \, dx \\ &= \int (2\csc^2 2x - 2\csc 2x \cot 2x - 1) \, dx \\ &= -\cot 2x + \csc 2x - x + c \end{aligned}$$

$$136) \int \frac{\sin^2 x}{\sin^3 x} dx$$

$$= \int \frac{2 \sin x \cos x}{\sin^3 x} dx = 2 \int \frac{\cos x}{\sin^2 x} dx = 2 \int \csc x \cot x = -2 \csc x + c$$

$$137) \int \frac{\sin^2 x}{\sin^2 x} dx$$

$$= \int \frac{2 \sin x \cos x}{\sin^2 x} dx = 2 \int \frac{\cos x}{\sin x} dx = 2 \ln |\sin x| + c$$

$$138) \int \sin^3 x \cos^3 x dx$$

$$= \int \sin^2 x \sin x \cos^3 x dx = \int \cos^3 x \sin x (1 - \cos^2 x) dx$$

$$= \int (\cos^3 x \sin x - \cos^5 x \sin x) dx = -\frac{1}{4} \cos^4 x + \frac{1}{6} \cos^6 x + c$$

$$\text{OR: } \int \sin^3 x \cos^3 x dx = \int \sin^3 x \cos^2 x \cos x dx$$

$$= \int (\sin^3 x \cos x - \sin^5 x \cos x) dx = \frac{1}{4} \sin^4 x - \frac{1}{6} \sin^6 x + c$$

$$\text{OR: } \int \sin^3 x \cos^3 x dx = \int (\sin x \cos x)^3 dx$$

$$= \frac{1}{8} \int \sin^3 2x dx = \frac{1}{8} \int (\sin 2x - \cos^2 2x \sin 2x) dx$$

$$= \frac{1}{8} \left[-\frac{1}{2} \cos 2x + \frac{1}{6} \cos^3 2x \right] + c = -\frac{1}{16} \cos 2x + \frac{1}{48} \cos^3 2x + c$$

$$139) \int \frac{1-\tan x}{1+\tan x} dx$$

$$= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx = \ln |\cos x + \sin x| + c$$

$$\text{OR: } \int \frac{1-\tan x}{1+\tan x} dx = \int \frac{\frac{\cos x}{\cos x + \sin x}}{\frac{\cos x}{\cos x + \sin x}} dx = \int \frac{\cos x - \sin x}{\cos x + \sin x} \cdot \frac{\cos x - \sin x}{\cos x - \sin x} dx$$

$$= \int \frac{\cos^2 x - 2\sin x \cos x + \sin^2 x}{\cos^2 x - \sin^2 x} dx = \int \frac{1 - \sin 2x}{\cos 2x} dx$$

$$= \int (\sec 2x) dx - \int \frac{\sin 2x}{\cos 2x} dx$$

$$= \frac{1}{2} \ln |\sec 2x + \tan 2x| + \frac{1}{2} \ln |\cos 2x| + c$$

$$140) \int \sin 2x \cot x dx$$

$$= \int 2 \sin x \cos x \cdot \frac{\cos x}{\sin x} dx = 2 \int \cos^2 x dx$$

$$= 2 \int \frac{1}{2} (1 + \cos 2x) dx = x + \frac{1}{2} \sin 2x + c$$

$$141) \int_1^3 \sqrt{x^2 - 4x + 4} dx$$

$$= \int_1^3 \sqrt{(x-2)^2} dx = \int_1^2 (2-x) dx + \int_2^3 (x-2) dx$$

$$= \left[2x - \frac{x^2}{2} \right]_1^2 + \left[\frac{x^2}{2} - 2x \right]_2^3 = \left[(4-2) - (2 - \frac{1}{2}) \right] + \left[\left(\frac{9}{2} - 6 \right) - (2-4) \right]$$

$$= \left[2 - \frac{3}{2} \right] + \left[-\frac{3}{2} + 2 \right] = \frac{1}{2} + \frac{1}{2} = 1$$

$$142) \int \sqrt{e^{2x-4}} dx$$

$$= \int \sqrt{e^{2(x-2)}} dx = \int \sqrt{(e^{x-2})^2} dx = \pm \int e^{x-2} dx = \pm e^{x-2} + c$$

$$143) \int (1 - 2e^{2x})^4 e^{2x} dx$$

$$= -\frac{1}{4} \int (1 - 2e^{2x})^4 (-4)e^{2x} dx = -\frac{1}{4} \cdot \frac{1}{5} (1 - 2e^{2x})^5 + c$$

$$= -\frac{1}{20} (1 - 2e^{2x})^5 + c$$

$$144) \int \sin 6x \cos^2 3x dx$$

$$= \int 2 \sin 3x \cos 3x \cos^2 3x dx = 2 \int \cos^3 3x \sin 3x dx$$

$$= -\frac{2}{3} \cdot \frac{1}{4} \cos^4 3x + c = -\frac{1}{6} \cos^4 3x + c$$

$$145) \int \frac{1}{(\cos x - \sin x)^2 (1 + \sin 2x)} dx$$

$$= \int \frac{1}{(\cos x - \sin x)^2 (\cos^2 x + 2 \sin x \cos x + \sin^2 x)} dx$$

$$= \int \frac{1}{(\cos x - \sin x)^2 (\cos x + \sin x)^2} dx = \int \frac{1}{[(\cos x - \sin x)(\cos x + \sin x)]^2} dx$$

$$= \int \frac{1}{(\cos^2 x - \sin^2 x)^2} dx = \int \frac{1}{\cos^4 x} dx = \int \sec^2 2x dx$$

$$= \frac{1}{2} \tan 2x + c$$

$$146) \int \sqrt[n]{x^3} dy = \sqrt[n]{x^3} y + c$$

$$147) \int \frac{\tan^3 x \sec^2 x}{\sec^4 x - 1} dx$$

$$\begin{aligned} &= \int \frac{(\sec^2 x - 1) \tan x \sec^2 x}{(\sec^2 x - 1)(\sec^2 x + 1)} dx = \frac{1}{2} \int \frac{2 \tan x \sec^2 x}{\sec^2 x + 1} dx \\ &= \frac{1}{2} \ln |\sec^2 x + 1| + c \end{aligned}$$

$$148) \int \frac{2 \sin^2 x + \cos^2 x}{\sec^2 x} dx$$

$$\begin{aligned} &= \int \frac{2 \sin^2 x + \cos^2 x - \sin^2 x}{\sec^2 x} dx = \int \frac{\sin^2 x + \cos^2 x}{\sec^2 x} dx \\ &= \int \frac{1}{\sec^2 x} dx = \int \cos^2 x dx = \frac{1}{2} \int (1 + \cos 2x) dx \\ &= \frac{1}{2} \left(x + \frac{1}{2} \sin 2x \right) + c = \frac{1}{2} x + \frac{1}{4} \sin 2x + c \end{aligned}$$

$$149) \int \frac{\sec^5 x \tan x}{\sqrt{\cos x}} dx$$

$$\begin{aligned} &= \int \sec^5 x \tan x \sec^2 x dx = \\ &\int \sec^{\frac{11}{2}} x \tan x dx = \int \sec^{\frac{9}{2}} x \sec x \tan x dx \\ &= \frac{2}{11} \sec^{\frac{11}{2}} x + c = \frac{2}{11} \sqrt{\sec^{11} x} + c \end{aligned}$$

$$150) \int \frac{1 - \cos 2x}{\sin 2x} dx$$

$$\begin{aligned} &= \int \left(\frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x} \right) dx = \int \left(\csc 2x \cdot \frac{\csc 2x + \cot 2x}{\csc 2x + \cot 2x} - \frac{\cos 2x}{\sin 2x} \right) dx \\ &= -\frac{1}{2} \ln |\csc 2x + \cot 2x| - \frac{1}{2} \ln |\sin 2x| + c \end{aligned}$$

$$151) \int \frac{\sin x + \cos x}{\cos^3 x} dx$$

$$\begin{aligned} &= \int \left(\frac{\sin x}{\cos^3 x} + \frac{\cos x}{\cos^3 x} \right) dx = \int (\tan x \sec^2 x + \sec^2 x) dx \\ &= \frac{1}{2} \tan^2 x + \tan x + c \end{aligned}$$

$$152) \int \frac{e^{\tan x} + \sin^2 x e^{\tan x}}{\sin^4 x - 1} dx$$

$$\begin{aligned} &= \int \frac{e^{\tan x} (1 + \sin^2 x)}{(\sin^2 x - 1)(\sin^2 x + 1)} dx = - \int \frac{e^{\tan x}}{\cos^2 x} dx \\ &= - \int e^{\tan x} \sec^2 x dx = -e^{\tan x} + c \end{aligned}$$

$$153) \int \tan^5 x dx$$

$$\begin{aligned} &= \int \tan^3 x \tan^2 x dx = \int \tan^3 x (\sec^2 x - 1) dx \\ &= \int (\tan^3 x \sec^2 x - \tan x (\sec^2 x - 1)) dx \\ &= \int (\tan^3 x \sec^2 x - \tan x \sec^2 x + \tan x) dx \\ &= \int \left(\tan^3 x \sec^2 x - \tan x \sec^2 + \frac{\sin x}{\cos x} \right) dx \\ &= \frac{1}{4} \tan^4 x - \frac{1}{2} \tan^2 x - \ln |\cos x| + c \end{aligned}$$

$$154) \int \sec^5 x \, dx$$

$$\begin{aligned} &= \int \sec^n x \, dx = \frac{1}{n} \tan x \sec^{n-2} x + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx \\ &= \frac{1}{4} \tan x \sec^3 x + \frac{3}{4} \int \sec^3 x \, dx \\ &= \frac{1}{4} \tan x \sec^3 x + \frac{3}{4} \left(\frac{1}{2} \tan x \sec x + \frac{1}{2} \int \sec x \, dx \right) \\ &= \frac{1}{4} \tan x \sec^3 x + \frac{3}{8} \tan x \sec x + \frac{3}{8} \ln |\sec x + \tan x| + c \end{aligned}$$

$$155) \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\tan x + \tan^3 x) \, dx$$

$$\begin{aligned} &= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan x (1 + \tan^2 x) \, dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan x \sec^2 x \, dx \\ &= \left[\frac{1}{2} \tan^2 x \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{1}{2} \left[\left(\tan \frac{\pi}{3} \right)^2 - \left(\tan \frac{\pi}{6} \right)^2 \right] \\ &= \frac{1}{2} \left[3 - \frac{1}{3} \right] = \frac{1}{2} \left(\frac{8}{3} \right) = \frac{4}{3} \end{aligned}$$

$$156) \int \frac{2\cos x - e^{-ix}}{\cos x + i \sin x} \, dx$$

$$\begin{aligned} &= \int \frac{2\cos x - (\cos x - i \sin x)}{\cos x + i \sin x} \, dx = \int \frac{2\cos x - \cos x + i \sin x}{\cos x + i \sin x} \, dx \\ &= \int \frac{\cos x + i \sin x}{\cos x + i \sin x} \, dx = \int dx = x + c \end{aligned}$$

$$157) \int \frac{\sqrt{\cot 2x}}{1-\cos^2 2x} dx$$

$$\begin{aligned} &= \int \frac{\sqrt{\cot 2x}}{\sin^2 2x} dx = \int \cot^{\frac{1}{2}} 2x \csc^2 2x dx \\ &= -\frac{1}{2} \cdot \frac{2}{3} \cot^{\frac{3}{2}} 2x + c = -\frac{1}{3} \sqrt{\cot^3 2x} + c \end{aligned}$$

$$158) \int \sqrt{\sec^2 x - 1} \tan x dx$$

$$\begin{aligned} &= \int \sqrt{\tan^2 x} \tan x dx = \pm \int \tan^2 x dx \\ &= \pm \int (\sec^2 x - 1) dx = \pm (\tan x - x) + c \end{aligned}$$

$$160) \int \frac{1-\cos x}{1+\cos x} dx$$

$$\begin{aligned} &= \int \frac{1-\cos x}{1+\cos x} \frac{1-\cos x}{1-\cos x} dx = \int \frac{1-2\cos x+\cos^2 x}{1-\cos^2 x} dx \\ &= \int \frac{1-2\cos x+\cos^2 x}{\sin^2 x} dx = \int (\csc^2 x - 2\csc x \cot x + \cot^2 x) dx \\ &= \int (\csc^2 x - 2\csc x \cot x + \csc^2 x - 1) dx \\ &= \int (2\csc^2 x - 2\csc x \cot x - 1) dx = -2\cot x + 2\csc x - x + c \end{aligned}$$

$$161) \int \sec^4 x dx$$

$$\begin{aligned} &= \int \sec^2 x \sec^2 x dx = \int (1 + \tan^2 x) \sec^2 x dx \\ &= \int (\sec^2 x + \tan^2 x \sec^2 x) dx = \tan x + \frac{1}{3} \tan^3 x + c \end{aligned}$$

$$162) \int \tan^4 x dx$$

$$\begin{aligned} &= \int \tan^2 x \tan^2 x dx = \int \tan^2 x (\sec^2 x - 1) dx \\ &= \int (\tan^2 x \sec^2 x - \tan^2 x) dx = \int (\tan^2 x \sec^2 x - \sec^2 x + 1) dx \\ &= \frac{1}{3} \tan^3 x - \tan x + x + C \end{aligned}$$

$$163) \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1-2\sin^3 x}{\sin^2 x} dx$$

$$\begin{aligned} &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1-2\sin x(1-\cos^2 x)}{\sin^2 x} dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1-2\sin x+2\cos^2 x \sin x}{\sin^2 x} dx \\ &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\csc^2 x - 2\csc x + 2\csc x \cot x) dx \\ &= [-\cot x + 2\ln|\csc x + \cot x| - 2\csc x]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \\ &= [-1 + 2\ln|\sqrt{2} + 1| - 2\sqrt{2}] - [1 + \ln|-1 - \sqrt{2}| - 2\sqrt{2}] \\ &= -1 + 2\ln(\sqrt{2} + 1) - 2\sqrt{2} - 1 - 2\ln(-1 - \sqrt{2}) + 2\sqrt{2} = -2 \end{aligned}$$

$$164) \int \frac{1}{\sin 2x \cos 2x} dx$$

$$\begin{aligned} &= \int \frac{\sin^2 2x + \cos^2 2x}{\sin 2x \cos 2x} dx = \int \left(\frac{\sin 2x}{\cos 2x} + \frac{\cos 2x}{\sin 2x} \right) dx \\ &= -\frac{1}{2} \ln|\cos 2x| + \frac{1}{2} \ln|\sin 2x| + C \end{aligned}$$

$$165) \int \frac{1}{\sec 2x \csc 2x} dx$$

$$= \int \sin 2x \cos 2x dx = \frac{1}{2} \int \sin 4x dx = -\frac{1}{8} \cos 4x + c$$

$$\text{OR: } 165) \int \frac{1}{\sec 2x \csc 2x} dx = \int \sin 2x \cos 2x dx$$

$$= \frac{1}{4} \sin^2 2x + c$$

$$166) \int \frac{\cos^2 x}{\cos^4 x} dx$$

$$= \int \frac{\cos^2 x - \sin^2 x}{\cos^4 x} dx = \int (\sec^2 x - \tan^2 x \sec^2 x) dx$$

$$= \tan x - \frac{1}{3} \tan^3 x + c$$

$$\text{OR: } \int \frac{\cos^2 x}{\cos^4 x} dx = \int \frac{2\cos^2 x - 1}{\cos^4 x} dx = \int (2\sec^2 x - \sec^4 x) dx$$

$$= \int (2\sec^2 x - \sec^2 x (\tan^2 x + 1)) dx$$

$$= \int (2\sec^2 x - \tan^2 x \sec^2 x - \sec^2 x) dx$$

$$= \int (\sec^2 x - \tan^2 x \sec^2 x) dx = \tan x - \frac{1}{3} \tan^3 x + c$$

$$167) \int (\sec 2x \csc 2x) dx$$

$$= \int \frac{1}{\sin 2x \cos 2x} dx = \int \frac{2}{2 \sin 2x \cos 2x} dx = 2 \int \frac{1}{\sin 4x} dx = 2 \int \csc 4x dx$$

$$= -\frac{1}{2} \ln |\csc 4x + \cot 4x| + c = \ln \left| \frac{1}{\sqrt{\csc 4x + \cot 4x}} \right| + c$$

$$168) \int (\sec 2x + \csc 2x) dx$$

$$\begin{aligned} &= \frac{1}{2} \ln |\sec 2x + \tan 2x| - \frac{1}{2} \ln |\csc 2x + \cot 2x| + c \\ &= \ln |\sqrt{\sec 2x + \tan 2x}| - \ln |\sqrt{\csc 2x + \cot 2x}| + c \\ &= \ln \left| \sqrt{\frac{\sec 2x + \tan 2x}{\csc 2x + \cot 2x}} \right| + c \end{aligned}$$

$$169) \int (\sec 2x - \csc 2x) dx$$

$$\begin{aligned} &= \frac{1}{2} \ln |\sec 2x + \tan 2x| + \frac{1}{2} \ln |\csc 2x + \cot 2x| + c \\ &= \ln |\sqrt{\sec 2x + \tan 2x}| + \ln |\sqrt{\csc 2x + \cot 2x}| + c \\ &= \ln \left| \sqrt{(\sec 2x + \tan 2x)(\csc 2x + \cot 2x)} \right| + c \end{aligned}$$

$$170) \int \frac{\sec 2x}{\csc 2x} dx$$

$$= \int \frac{\frac{1}{\cos 2x}}{\frac{1}{\sin 2x}} dx = \int \frac{\sin 2x}{\cos 2x} dx = -\frac{1}{2} \ln |\cos 2x| + c$$

$$171) \int \frac{\csc 2x}{\sec 2x} dx$$

$$= \int \frac{\frac{1}{\sin 2x}}{\frac{1}{\cos 2x}} dx = \int \frac{\cos 2x}{\sin 2x} dx = \frac{1}{2} \ln |\sin 2x| + c$$

$$172) \int (\csc 2x - \sec 2x) dx$$

$$= -\frac{1}{2} \ln |\csc 2x + \cot 2x| - \frac{1}{2} \ln |\sec 2x + \tan 2x| + c$$

$$173) \int \frac{e^{3x}}{\ln 3 + e^{3x}} dx$$

$$= \frac{1}{3} \ln |\ln 3 + e^{3x}| + c$$

$$174) \int \frac{e^{\sin^2 x}}{\csc 2x} dx$$

$$= \int e^{\sin^2 x} \cdot \sin 2x dx = \int e^{\sin^2 x} (2 \sin x \cos x) dx = e^{\sin^2 x} + c$$

$$175) \int \frac{\sin^3 x}{\cos x - \cos^2 x} dx$$

$$= \int \frac{(1-\cos^2 x) \sin x}{\cos x(1-\cos x)} dx = \int \frac{(1-\cos x)(1+\cos x) \sin x}{\cos x(1-\cos x)} dx$$

$$= \int \frac{(1+\cos x) \sin x}{\cos x} dx = \int \frac{\sin x}{\cos x} dx + \int \sin x dx$$

$$= -\ln |\cos x| - \cos x + c$$

$$176) \int \sqrt{\sin x} \cos^3 x dx$$

$$= \int (\sin x)^{\frac{1}{2}} (1 - \sin^2 x) \cos x dx$$

$$= \int \left(\sin^{\frac{1}{2}} x \cos x - \sin^{\frac{5}{2}} x \cos x \right) dx$$

$$= \frac{2}{3} \sin^{\frac{3}{2}} x - \frac{2}{7} \sin^{\frac{7}{2}} x + c = \frac{2}{3} \sqrt{\sin^3 x} - \frac{2}{7} \sqrt{\sin^7 x} + c$$

$$177) \int_0^{\frac{\pi}{16}} \sec^2 4x \, dx$$

$$= \left[\frac{1}{4} \tan 4x \right]_0^{\frac{\pi}{16}} = \frac{1}{4} \left(\tan \frac{\pi}{4} - \tan 0 \right) = \frac{1}{4} (1) = \frac{1}{4}$$

$$178) \int \sin^4 x \cos^4 x \, dx$$

$$\begin{aligned} &= \frac{1}{16} \int 16 \sin^4 x \cos^4 x \, dx = \frac{1}{16} \int \sin^4 2x \, dx \\ &= \frac{1}{16} \int \left(\frac{1}{2} - \frac{1}{2} \cos 4x \right)^2 \, dx = \frac{1}{16} \int \left(\frac{1}{4} - \frac{1}{2} \cos 4x + \frac{1}{4} \cos^2 4x \right) \, dx \\ &= \frac{1}{16} \int \left(\frac{1}{4} - \frac{1}{2} \cos 4x + \frac{1}{8} + \frac{1}{8} \cos 8x \right) \, dx \\ &= \frac{1}{16} \left(\frac{3}{8}x - \frac{1}{8} \sin 4x + \frac{1}{64} \sin 8x \right) + c \end{aligned}$$

$$179) \int \frac{\ln^2 x - 1}{x \ln x + x} \, dx$$

$$\begin{aligned} &= \int \frac{(\ln x - 1)(\ln x + 1)}{x(\ln x + 1)} \, dx = \int \frac{\ln x - 1}{x} \, dx \\ &= \int \frac{\ln x}{x} \, dx - \int \frac{1}{x} \, dx = \frac{1}{2} \ln^2 x - \ln|x| + c \end{aligned}$$

$$OR: \int \frac{\ln x - 1}{x} \, dx = \frac{1}{2} (\ln x - 1)^2 + c$$

$$180) \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{dx}{\sin^2 2x}$$

$$\begin{aligned} &= \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \csc^2 2x \, dx = \left[-\frac{1}{2} \cot 2x \right]_{\frac{\pi}{8}}^{\frac{\pi}{4}} \\ &= -\frac{1}{2} \left[\cot \frac{\pi}{2} - \cot \frac{\pi}{4} \right] = -\frac{1}{2} (0 - 1) = \frac{1}{2} \end{aligned}$$

$$181) \int \frac{1}{1+\cot^2 x} dx$$

$$\begin{aligned} &= \int \frac{1}{\csc^2 x} dx = \int \sin^2 x dx = \frac{1}{2} \int (1 - \cos 2x) dx \\ &= \frac{1}{2} \left(x - \frac{1}{2} \sin 2x \right) + c = \frac{1}{2} x - \frac{1}{4} \sin 2x + c \end{aligned}$$

$$182) \int_0^{\frac{\pi}{2}} \sin^2 x dx$$

$$\begin{aligned} &= \int_0^{\frac{\pi}{2}} \frac{1}{2} (1 - \cos 2x) dx = \left[\frac{1}{2} x - \frac{1}{4} \sin 2x \right]_0^{\frac{\pi}{2}} \\ &= \left(\frac{\pi}{4} - \frac{1}{4} \sin \pi \right) - \left(0 - \frac{1}{4} \sin 0 \right) = \frac{\pi}{4} \end{aligned}$$

$$183) \int \frac{\cos^4 x}{\cos x} dx$$

$$\begin{aligned} &= \int \frac{1 - 2\sin^2 2x}{\cos x} dx = \int \frac{1 - 2(2\sin x \cos x)^2}{\cos x} dx \\ &= \int \frac{1 - 8\sin^2 x \cos^2 x}{\cos x} dx = \int (\sec x - 8\sin^2 x \cos x) dx \\ &= \ln|\sec x + \tan x| - \frac{8}{3} \sin^3 x + c \end{aligned}$$

$$184) \int \frac{2\sin^3 \sqrt{x}}{\sqrt[3]{x^2}} dx$$

$$= 2(3) \int \sin^3 \sqrt{x} \cdot \frac{1}{3\sqrt[3]{x^2}} dx = -6\cos^3 \sqrt{x} + c$$

$$185) \int \tan^7 x dx$$

$$\begin{aligned} &= \int \tan^5 x \tan^2 x dx = \int \tan^5 x (\sec^2 x - 1) dx \\ &= \int (\tan^5 x \sec^2 x - \tan^5 x) dx \\ &= \int (\tan^5 x \sec^2 x - \tan^3 x (\sec^2 x - 1)) \\ &= \int (\tan^5 x \sec^2 x - \tan^3 x \sec^2 x + \tan^3 x) \\ &= \int (\tan^5 x \sec^2 x - \tan^3 x \sec^2 x + \tan x \sec^2 x - \tan x) dx \\ &= \frac{1}{6} \tan^6 x - \frac{1}{4} \tan^4 x + \frac{1}{2} \tan^2 x + \ln |\cos x| + c \end{aligned}$$

$$186) \int_0^\pi \sqrt{1 - \sin 2x} dx$$

$$\begin{aligned} &= \int_0^\pi \sqrt{\cos^2 x - 2 \sin x \cos x + \sin^2 x} dx = \int_0^\pi \sqrt{(\cos x - \sin x)^2} dx \\ &= \int_0^\pi |\cos x - \sin x| dx, \cos x - \sin x = 0 \rightarrow \tan x = 1 \rightarrow x = \frac{\pi}{4} \in [0, \pi] \\ &= \int_0^\pi |\cos x - \sin x| dx = \int_0^{\frac{\pi}{4}} (\cos x - \sin x) dx + \int_{\frac{\pi}{4}}^\pi (\sin x - \cos x) dx \\ &= [\sin x + \cos x]_0^{\frac{\pi}{4}} + [-\cos x - \sin x]_{\frac{\pi}{4}}^\pi \\ &= \left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) - (0 + 1) \right] + \left[(-(-1) - 0) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right) \right] \\ &= (\sqrt{2} - 1) + (1 + \sqrt{2}) = 2\sqrt{2} \end{aligned}$$

$$187) \int \frac{x^2 - 4x + 3}{x^2 - 2x + 1} dx$$

$$\begin{aligned} &= \int \frac{(x-3)(x-1)}{(x-1)(x-1)} dx = \int \frac{(x-3)}{(x-1)} dx = \int \frac{x}{x-1} dx - 3 \int \frac{1}{x-1} dx \\ &= \int \frac{x-1+1}{x-1} dx - 3 \int \frac{1}{x-1} dx = \int dx + \int \frac{1}{(x-1)} dx - 3 \int \frac{1}{(x-1)} dx \\ &= \int dx - 2 \int \frac{1}{(x-1)} dx = x - 2 \ln|x-1| + c \end{aligned}$$

$$OR: \int \frac{x^2 - 4x + 3}{x^2 - 2x + 1} dx = \int \frac{x^2 - 2x + 1 - 2x + 2}{x^2 - 2x + 1} dx$$

$$= \int \frac{x^2 - 2x + 1}{x^2 - 2x + 1} dx - \int \frac{2x - 2}{x^2 - 2x + 1} dx = x - \ln|x^2 - 2x + 1| + c$$

$$188) \int \tan^3 x dx$$

$$\begin{aligned} &= \int \tan^2 x \tan x dx = \int \tan x (\sec^2 x - 1) dx \\ &= \int (\tan x \sec^2 x - \tan x) dx = \frac{1}{2} \tan^2 x + \ln|\cos x| + c \end{aligned}$$

$$189) \int \cot x \sec^2 x dx$$

$$\begin{aligned} &= \int \frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x} dx = \int \frac{1}{\sin x \cos x} dx = \int \frac{2}{2 \sin x \cos x} dx \\ &= 2 \int \frac{1}{\sin 2x} dx = 2 \int \csc 2x dx = -\ln|\csc 2x + \cot 2x| + c \end{aligned}$$

$$190) \int \tan^{-3} x \sec^2 x dx = -\frac{1}{2} \tan^{-2} x + c = -\frac{1}{2 \tan^2 x} + c$$

$$191) \int \cos y \cos 5y dy$$

$$\begin{aligned} &= \frac{1}{2} \int (\cos 5y + \cos 3y) dy = \frac{1}{2} \left(\frac{1}{5} \sin 5y + \frac{1}{3} \sin 3y \right) + c \\ &= \frac{1}{10} \sin 5y + \frac{1}{6} \sin 3y + c \end{aligned}$$

$$192) \int \sin 4y \cos 5y dy$$

$$\begin{aligned} &= \int \frac{1}{2} (\sin 9y + \sin y) dy = \frac{1}{2} \left(-\frac{1}{9} \cos 9y - \cos y \right) + c \\ &= -\frac{1}{18} \cos 9y - \frac{1}{2} \cos y + c \end{aligned}$$

$$193) \int \sec^4 \theta d\theta$$

$$\begin{aligned} &= \int \sec^2 \theta \sec^2 \theta d\theta = \int \sec^2 \theta (\tan^2 \theta + 1) d\theta \\ &= \int (\tan^2 \theta \sec^2 \theta + \sec^2 \theta) d\theta = \frac{1}{3} \tan^3 \theta + \tan \theta + c \end{aligned}$$

$$194) \int \frac{x^3}{(x+1)^5} dx$$

$$\begin{aligned} &= \int \frac{x^3}{(x+1)^3(x+1)^2} dx = \int \left(\frac{x}{x+1} \right)^3 \cdot \frac{1}{(x+1)^2} dx \\ &= \frac{1}{4} \left(\frac{x}{x+1} \right)^4 + c \end{aligned}$$

$$195) \int \tan^3 3x \sec^3 3x dx$$

$$\begin{aligned} &= \int \tan^2 3x \tan 3x \sec^3 3x dx = \int (\sec^2 3x - 1) \tan 3x \sec^3 3x dx \\ &= \int (\sec^5 3x \tan 3x - \sec^3 3x \tan 3x) dx \\ &= \int (\sec^4 3x \sec 3x \tan 3x - \sec^2 3x \sec 3x \tan 3x) dx \\ &= \frac{1}{15} \sec^5 3x - \frac{1}{9} \sec^3 3x + c \end{aligned}$$

$$196) \int \frac{\cot(7x+5)\sin(3x+2)}{\csc(7x+5)} dx$$

$$\begin{aligned} &= \int \frac{\cos(7x+5)}{\sin(7x+5)} \cdot \sin(7x+5) \sin(3x+2) dx \\ &= \int \cos(7x+5) \cdot \sin(3x+2) dx \\ &= \frac{1}{2} \int [\sin(10x+7) + \sin(4x+3)] dx \\ &= \frac{1}{2} \left[-\frac{1}{10} \cos(10x+7) - \frac{1}{4} \cos(4x+3) \right] + c \\ &= -\frac{1}{20} \cos(10x+7) - \frac{1}{8} \cos(4x+3) + c \end{aligned}$$

$$197) \int (\sin 2x + \cos 2x)^2 dx$$

$$\begin{aligned} &= \int (\sin^2 2x + 2 \sin 2x \cos 2x + \cos^2 2x) dx \\ &= \int (1 + \sin 4x) dx = x - \frac{1}{4} \cos 4x + c \end{aligned}$$

$$198) \int (\tan x + \sec x)^2 dx$$

$$= \int (\tan^2 x + 2\tan x \sec x + \sec^2 x) dx$$

$$= \int (\sec^2 x - 1 + 2\tan x \sec x + \sec^2 x) dx$$

$$= \int (2\sec^2 x - 1 + 2\tan x \sec x) dx = 2\tan x + 2\sec x - x + c$$

$$199) \int \sin x \cos \frac{x}{2} dx$$

$$= \int 2\sin \frac{x}{2} \cos \frac{x}{2} \cos \frac{x}{2} dx = 2 \int \cos^2 \frac{x}{2} \sin \frac{x}{2} dx$$

$$= -4 \cdot \frac{1}{3} \cos^3 \frac{x}{2} + c = -\frac{4}{3} \cos^3 \frac{x}{2} + c$$

$$200) \int \frac{2\sin 2x}{\sqrt{\cos x}} dx$$

$$= \int \frac{4\sin x \cos x}{\sqrt{\cos x}} dx = 4 \int \frac{\sin x \sqrt{\cos x} \sqrt{\cos x}}{\sqrt{\cos x}} dx = 4 \int \sqrt{\cos x} \sin x dx$$

$$= 4 \int (\cos x)^{\frac{1}{2}} \sin x dx = 4 \left(-\frac{2}{3} \right) (\cos x)^{\frac{3}{2}} + c = -\frac{8}{3} \sqrt{\cos^3 x} + c$$

$$201) \int \cos^2 \frac{x}{3} dx$$

$$= \int \frac{1}{2} \left(1 + \cos \frac{2x}{3} \right) dx = \frac{1}{2} \left(x + \frac{3}{2} \sin \frac{2x}{3} \right) + c$$

$$= \frac{x}{2} + \frac{3}{4} \sin \frac{2x}{3} + c$$

202) $\int \cos^3 2x \sin x dx$

$$\begin{aligned}
 &= \int (\cos 2x)^3 \sin x dx = \int (2\cos^2 x - 1)^3 \sin x dx \\
 &= \int (4\cos^4 x - 4\cos^2 x + 1)(2\cos^2 x - 1) \sin x dx \\
 &= \int (8\cos^6 x - 8\cos^4 x + 2\cos^2 x - 4\cos^4 x + 4\cos^2 x - 1) \sin x dx \\
 &= \int (8\cos^6 x - 12\cos^4 x + 6\cos^2 x - 1) \sin x dx \\
 &= \int (8\cos^6 x \sin x - 12\cos^4 x \sin x + 6\cos^2 x \sin x - \sin x) dx \\
 &= -\frac{8}{7} \cos^7 x + \frac{12}{5} \cos^5 x - 2\cos^3 x + \cos x + c
 \end{aligned}$$

203) $\int \frac{\sec^4 x}{\sqrt{\tan x}} dx$

$$\begin{aligned}
 &= \int \tan^{-\frac{1}{2}} x \sec^2 x \sec^2 x dx = \int \tan^{-\frac{1}{2}} x \sec^2 x (\tan^2 x + 1) dx \\
 &= \int \left(\tan^{\frac{3}{2}} x \sec^2 x + \tan^{-\frac{1}{2}} x \sec^2 x \right) dx = \frac{2}{5} \tan^{\frac{5}{2}} x + 2\tan^{\frac{1}{2}} x + c \\
 &= \frac{2}{5} \sqrt{\tan^5 x} + 2\sqrt{\tan x} + c
 \end{aligned}$$

204) $\int \frac{\cos^3 x}{\sqrt{\sin x}} dx$

$$\begin{aligned}
 &= \int \sin^{-\frac{1}{2}} x \cos x \cos^2 x dx = \int \sin^{-\frac{1}{2}} x \cos x (1 - \sin^2 x) dx \\
 &= \int \left(\sin^{-\frac{1}{2}} x \cos x - \sin^{\frac{3}{2}} x \cos x \right) dx = 2\sin^{\frac{1}{2}} x - \frac{2}{5} \sin^{\frac{5}{2}} x + c \\
 &= 2\sqrt{\sin x} - \frac{2}{5} \sqrt{\sin^5 x} + c
 \end{aligned}$$

205) $\int \sin^2 x \cos^4 x dx$

$$\begin{aligned}
 &= \int \left(\frac{1}{2} - \frac{1}{2} \cos 2x \right) \left(\frac{1}{2} + \frac{1}{2} \cos 2x \right)^2 dx \\
 &= \int \left(\frac{1}{2} - \frac{1}{2} \cos 2x \right) \left(\frac{1}{4} + \frac{1}{2} \cos 2x + \frac{1}{4} \cos^2 2x \right) dx \\
 &= \int \left(\frac{1}{8} + \frac{1}{4} \cos 2x + \frac{1}{8} \cos^2 2x - \frac{1}{8} \cos 2x - \frac{1}{4} \cos^2 2x - \frac{1}{8} \cos^3 2x \right) dx \\
 &= \int \left(\frac{1}{8} + \frac{1}{8} \cos 2x - \frac{1}{8} \cos^2 2x - \frac{1}{8} \cos^3 2x \right) dx \\
 &= \int \left(\frac{1}{8} + \frac{1}{8} \cos 2x - \frac{1}{8} \left(\frac{1}{2} + \frac{1}{2} \cos 4x \right) - \frac{1}{8} (1 - \sin^2 2x) \cos 2x \right) dx \\
 &= \int \left(\frac{1}{8} + \frac{1}{8} \cos 2x - \frac{1}{16} - \frac{1}{16} \cos 4x - \frac{1}{8} \cos 2x + \frac{1}{8} \sin^2 2x \cos 2x \right) dx \\
 &= \int \left(\frac{1}{16} - \frac{1}{16} \cos 4x + \frac{1}{8} \sin^2 2x \cos 2x \right) dx \\
 &= \frac{x}{16} - \frac{1}{64} \sin 4x + \frac{1}{48} \sin^3 2x + c
 \end{aligned}$$

206) $\int \sin^3 x \cos^2 x dx$

$$\begin{aligned}
 &= \int \cos^2 x \sin x (1 - \cos^2 x) dx = \int (\cos^2 x \sin x - \cos^4 x \sin x) dx \\
 &= -\frac{1}{3} \cos^3 x + \frac{1}{5} \cos^5 x + c
 \end{aligned}$$

$$207) \int \sec^3 x \tan^3 x dx$$

$$\begin{aligned} &= \int \sec^3 x \tan x \tan^2 x dx = \int \sec^3 x \tan x (\sec^2 x - 1) dx \\ &= \int (\sec^5 x \tan x - \sec^3 x \tan x) \\ &= \int (\sec^4 x \sec x \tan x - \sec^2 x \sec x \tan x) dx \\ &= \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + c \end{aligned}$$

$$208) \int \tan^5 x \sec^{\frac{-3}{2}} x dx$$

$$\begin{aligned} &= \int \tan^3 x (\sec^2 x - 1) \sec^{\frac{-3}{2}} x dx \\ &= \int \left(\tan^3 x \sec^{\frac{1}{2}} x - \tan^3 x \sec^{\frac{-3}{2}} x \right) dx \\ &= \int \left[(\sec^2 x - 1) \tan x \sec^{\frac{1}{2}} x - (\sec^2 x - 1) \tan x \sec^{\frac{-3}{2}} x \right] dx \\ &= \int \left[\sec^{\frac{3}{2}} x \sec x \tan x - 2 \sec^{\frac{-1}{2}} x \sec x \tan x + \sec^{\frac{-5}{2}} x \sec x \tan x \right] dx \\ &= \frac{2}{5} \sec^{\frac{5}{2}} x - 4 \sec^{\frac{1}{2}} x - \frac{2}{3} \sec^{\frac{-3}{2}} x + c \\ &= \frac{2}{5} \sqrt{\sec^5 x} - 4 \sqrt{\sec x} - \frac{2}{3} \frac{1}{\sqrt{\sec^3 x}} + c \end{aligned}$$

$$209) \int \sin x \cos x dx$$

$$= \frac{1}{2} \sin^2 x + c , OR: \frac{-1}{2} \cos^2 x + c$$

$$210) \int_0^{\frac{\pi}{2}} \cos^2 x \cos 2x dx$$

$$= \int_0^{\frac{\pi}{2}} \cos^2 x (1 - 2\sin^2 x) dx$$

$$= \int_0^{\frac{\pi}{2}} (\cos^2 x - 2\sin^2 x \cos^2 x) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{1}{2} \cos 2x - \frac{1}{2} (4\sin^2 x \cos^2 x) \right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{1}{2} \cos 2x - \frac{1}{2} (\sin^2 2x) \right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{1}{2} \cos 2x - \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \cos 4x \right) \right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{1}{2} \cos 2x - \frac{1}{4} + \frac{1}{4} \cos 4x \right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1}{4} + \frac{1}{2} \cos 2x + \frac{1}{4} \cos 4x \right) dx$$

$$= \left[\frac{x}{4} + \frac{1}{4} \sin 2x + \frac{1}{16} \sin 4x \right]_0^{\frac{\pi}{2}} = \left(\frac{\pi}{8} + 0 + 0 \right) - (0) = \frac{\pi}{8}$$

$$211) \int \sin 2x \cos x dx$$

$$= \int 2 \sin x \cos x \cos x dx$$

$$= 2 \int \cos^2 x \sin x dx$$

$$= -\frac{2}{3} \cos^3 x + c$$

$$\begin{aligned}
 212) & \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \sin^2 x \cos 2x dx \\
 &= \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \sin^2 x (2\cos^2 x - 1) dx \\
 &= \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} (2\sin^2 x \cos^2 x - \sin^2 x) dx \\
 &= \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \left(\frac{1}{2} (4\sin^2 x \cos^2 x) - \left(\frac{1}{2} - \frac{1}{2} \cos 2x \right) \right) dx \\
 &= \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \left(\frac{1}{2} \sin^2 2x - \frac{1}{2} + \frac{1}{2} \cos 2x \right) dx \\
 &= \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \left(\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \cos 4x \right) - \frac{1}{2} + \frac{1}{2} \cos 2x \right) dx \\
 &= \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \left(\frac{1}{4} - \frac{1}{4} \cos 4x - \frac{1}{2} + \frac{1}{2} \cos 2x \right) dx \\
 &= \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \left(\frac{-1}{4} - \frac{1}{4} \cos 4x + \frac{1}{2} \cos 2x \right) dx \\
 &= \left[-\frac{x}{4} - \frac{1}{16} \sin 4x + \frac{1}{4} \sin 2x \right]_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \\
 &= \left[-\frac{3\pi}{4} - 0 - \frac{1}{4} \right] - \left[0 - 0 - \frac{\pi}{2} \right] = \frac{-4 - 3\pi + 8\pi}{16} = \frac{5\pi - 4}{16}
 \end{aligned}$$

$$213) \int \sqrt{\tan x} \sec^4 x \, dx$$

$$\begin{aligned} &= \int \tan^{\frac{1}{2}} x \sec^2 x \sec^2 x \, dx = \int \tan^{\frac{1}{2}} x \sec^2 x (\tan^2 x + 1) \, dx \\ &= \int \left(\tan^{\frac{5}{2}} x \sec^2 x + \tan^{\frac{1}{2}} x \sec^2 x \right) \, dx \\ &= \frac{2}{7} \tan^{\frac{7}{2}} x + \frac{2}{3} \tan^{\frac{3}{2}} x + c = \frac{2}{7} \sqrt{\tan^7 x} + \frac{2}{3} \sqrt{\tan^3 x} + c \end{aligned}$$

$$214) \int \cos 2x \sin x \, dx$$

$$\begin{aligned} &= \int (2\cos^2 x - 1) \sin x \, dx = \int (2\cos^2 x \sin x - \sin x) \, dx \\ &= -\frac{2}{3} \cos^3 x + \cos x + c \end{aligned}$$

$$215) \int \cos 2x \sin 2x \, dx$$

$$= \frac{1}{4} \sin^2 2x + c , OR: -\frac{1}{4} \cos^2 2x + c$$

$$216) \int \sin^3 x \cos^2 x \, dx$$

$$\begin{aligned} &= \int \sin^2 x \sin x \cos^2 x \, dx \\ &= \int (1 - \cos^2 x) \sin x \cos^2 x \, dx \\ &= \int (\cos^2 x \sin x - \cos^4 x \sin x) \, dx \\ &= -\frac{1}{3} \cos^3 x + \frac{1}{5} \cos^5 x + c \end{aligned}$$

$$217) \int \frac{\cos x}{\sin^4 x} dx$$

$$= \int \sin^{-4} x \cos x dx = -\frac{1}{3} \sin^{-3} x + c$$

$$= \frac{-1}{3 \sin^3 x} + c = -\frac{1}{3} \csc^3 x + c$$

$$\text{OR: } \int \frac{\cos x}{\sin^4 x} dx = \int \csc^3 x \cot x dx = \int \csc^2 x \csc x \cot x dx$$

$$= -\frac{1}{3} \csc^3 x + c$$

$$218) \int \frac{\sin^3 x - 4}{1 - \cos^2 x} dx$$

$$= \int \frac{\sin^3 x}{1 - \cos^2 x} dx - \int \frac{4}{1 - \cos^2 x} dx$$

$$= \int \frac{\sin^3 x}{\sin^2 x} dx - \int \frac{4}{\sin^2 x} dx = \int \sin x dx - 4 \int \csc^2 x dx$$

$$= -\cos x + 4 \cot x + c$$

$$219) \int \cos^4 x \tan x dx$$

$$= \int \cos^4 x \cdot \frac{\sin x}{\cos x} dx = \int \cos^3 x \sin x dx = -\frac{1}{4} \cos^4 x + c$$

$$220) \int \sec^4 x \tan x dx$$

$$= \int \sec^3 x \sec x \tan x dx = \frac{1}{4} \sec^4 x + c$$

$$221) \int \frac{\sin 3x}{\sqrt[3]{\cos^4 3x}} dx$$

$$\begin{aligned} &= \int (\cos 3x)^{-\frac{4}{3}} \sin 3x dx = -\frac{1}{3} \cdot (-3)(\cos 3x)^{-\frac{1}{3}} + c \\ &= \frac{1}{(\cos 3x)^{\frac{1}{3}}} + c = \frac{1}{\sqrt[3]{\cos 3x}} + c \end{aligned}$$

$$222) \int \frac{\tan^3 x}{\cos^2 x} dx$$

$$= \int \tan^3 x \sec^2 x dx = \frac{1}{4} \tan^4 x + c$$

$$223) \int \frac{\cos^3 x}{\sin^4 x} dx$$

$$\begin{aligned} &= \int \cot^3 x \csc x dx = \int \cot^2 x \cot x \csc x dx \\ &= \int (\csc^2 x - 1) \cot x \csc x dx = \int (\csc^2 x \csc x \cot x - \csc x \cot x) dx \\ &= -\frac{1}{3} \csc^3 x + \csc x + c \end{aligned}$$

$$224) \int \frac{\sin x}{1 - \sin^2 x} dx$$

$$= \int \frac{\sin x}{\cos^2 x} dx = \int \sec x \tan x dx = \sec x + c$$

$$225) \int \sin 2x \sin^2 x dx$$

$$= \int 2 \sin x \cos x \sin^2 x dx = 2 \int \sin^3 x \cos x dx = \frac{1}{2} \sin^4 x + c$$

$$226) \int \frac{\sec x}{\cot x} dx$$

$$= \int \sec x \tan x dx = \sec x + c$$

$$227) \int \frac{\cos 2x + 2 \sin^2 x}{\cos^2 x} dx$$

$$= \int \frac{\cos^2 x - \sin^2 x + 2 \sin^2 x}{\cos^2 x} dx = \int \frac{\cos^2 x + \sin^2 x}{\cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx = \int \sec^2 x dx = \tan x + c$$

$$228) \int \frac{\sin 2x}{\sin x} dx$$

$$= \int \frac{2 \sin x \cos x}{\sin x} dx = 2 \int \cos x dx = 2 \sin x + c$$

$$229) \int \sin 6x \cos^2 3x dx$$

$$= \int 2 \sin 3x \cos 3x \cos^2 3x dx = 2 \int \cos^3 3x \sin 3x dx$$

$$= -\frac{2}{3} \cdot \frac{1}{4} \cos^4 3x + c$$

$$= -\frac{1}{6} \cos^4 3x + c$$

$$\begin{aligned}
 230) \quad & \int \frac{\cos^3 x + \sin^3 x}{\cos x + \sin x} dx \\
 &= \int \frac{(\cos x + \sin x)(\cos^2 x - \cos x \sin x + \sin^2 x)}{\cos x + \sin x} dx \\
 &= (\cos^2 x - \cos x \sin x + \sin^2 x) dx = \int (1 - \sin x \cos x) dx \\
 &= x - \frac{1}{2} \sin^2 x + c
 \end{aligned}$$

$$\begin{aligned}
 231) \quad & \int \sin 2x (\cos x + \sin x) dx \\
 &= \int 2 \sin x \cos x (\sin x + \cos x) dx \\
 &= 2 \int (\sin^2 x \cos x + \cos^2 x \sin x) dx \\
 &= 2 \left[\frac{1}{3} \sin^3 x - \frac{1}{3} \cos^3 x \right] + c = \frac{2}{3} \sin^3 x - \frac{2}{3} \cos^3 x + c
 \end{aligned}$$

$$\begin{aligned}
 232) \quad & \int \frac{\sin^3 x}{\cos^4 x} dx \\
 &= \int \tan^3 x \sec x dx = \int \tan^2 x \sec x \tan x dx \\
 &= \int (\sec^2 x - 1) \sec x \tan x dx \\
 &= \int (\sec^2 x \sec x \tan x - \sec x \tan x) dx = \frac{1}{3} \sec^3 x - \sec x + c
 \end{aligned}$$

$$233) \int \frac{\cos x + \sin x}{\sqrt[4]{\cos x - \sin x}} dx \\ = \int (\cos x - \sin x)^{-\frac{1}{4}} (\cos x + \sin x) dx$$

$$= -\frac{4}{3} (\cos x - \sin x)^{\frac{3}{4}} + c = -\frac{4}{3} \sqrt[4]{(\cos x - \sin x)^3} + c$$

$$234) \int \frac{\cos^2 x}{1 + \sin x} dx$$

$$= \int \frac{1 - \sin^2 x}{1 + \sin x} dx = \int \frac{(1 - \sin x)(1 + \sin x)}{1 + \sin x} dx = \int (1 - \sin x) dx$$

$$= x + \cos x + c$$

$$\text{OR: } \int \frac{\cos^2 x}{1 + \sin x} dx = \int \frac{\cos^2 x}{1 + \sin x} \cdot \frac{1 - \sin x}{1 - \sin x} dx$$

$$= \int \frac{\cos^2 x (1 - \sin x)}{1 - \sin^2 x} dx = \int \frac{\cos^2 x (1 - \sin x)}{\cos^2 x} dx$$

$$= \int (1 - \sin x) dx = x + \cos x + c$$

$$235) \int (\sin^2 2x - \cos x) dx$$

$$= \int \left(\frac{1}{2} - \frac{1}{2} \cos 4x - \cos 2x \right) dx$$

$$= \frac{x}{2} - \frac{1}{8} \sin 4x - \frac{1}{2} \sin 2x + c$$

$$236) \int \frac{\sin 3x}{\sec^2 3x} dx$$

$$= \int \cos^2 3x \sin 3x dx = -\frac{1}{9} \cos^3 3x + c$$

$$237) \int (1 + 2\sin 4x)^2 \cos 4x dx$$

$$= \frac{1}{24} (1 + 2\sin 4x)^3 + c$$

$$238) \int \frac{\sin 2x}{\sqrt{1+2\sin^2 x}} dx$$

$$= \int (1 + 2\sin^2 x)^{-\frac{1}{2}} \sin 2x dx = \int (1 + 2\sin^2 x)^{-\frac{1}{2}} 2\sin x \cos x dx$$

$$= \sqrt{1 + 2\sin^2 x} + c$$

$$239) \int \sqrt{1 + \cos 2x} \sin 2x dx$$

$$= \int (1 + \cos 2x)^{\frac{1}{2}} \sin 2x dx = -\frac{1}{2} \cdot \frac{2}{3} (1 + \cos 2x)^{\frac{3}{2}} + c$$

$$= -\frac{1}{3} \sqrt{\cos^3 2x} + c$$

$$240) \int \sqrt{1 - \cos 2x} \cos x dx$$

$$= \int \sqrt{1 - 1 + 2\sin^2 x} \cos x dx = \int \sqrt{2\sin^2 x} \cdot \cos x dx$$

$$= \pm \sqrt{2} \int \sin x \cos x dx = \pm \frac{1}{\sqrt{2}} \sin^2 x + c$$

$$241) \int \frac{\csc^2 3x}{\sqrt{1+2\cot 3x}} dx$$

$$= \int (1 + 2\cot 3x)^{-\frac{1}{2}} \csc^2 3x dx$$

$$= -\frac{1}{6} \cdot 2(1 + 2\cot 3x)^{\frac{1}{2}} + c = -\frac{1}{3}\sqrt{1 + 2\cot 3x} + c$$

$$242) \int \frac{\sin^2 2x}{1+\cos 2x} dx$$

$$= \int \frac{1-\cos^2 2x}{1+\cos 2x} dx = \int \frac{(1-\cos 2x)(1+\cos 2x)}{1+\cos 2x} dx$$

$$= \int (1 - \cos 2x) = x - \frac{1}{2}\sin 2x + c$$

$$243) \int \frac{\sqrt{\tan x + 1}}{\cos^2 x} dx$$

$$= \int (\tan x + 1)^{\frac{1}{2}} \sec^2 x dx = \frac{2}{3}(\tan x + 1)^{\frac{3}{2}} + c = \frac{2}{3}\sqrt{(\tan x + 1)^3} + c$$

$$244) \int (1 + \cos x)^2 dx$$

$$= \int (1 + 2\cos x + \cos^2 x) dx = \int \left(1 + 2\cos x + \frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$$

$$= \frac{3}{2}x + 2\sin x + \frac{1}{4}\sin 2x + c$$

$$245) \int (\cos x - \sin 2x)^2 dx$$

$$\begin{aligned} &= \int (\cos^2 x - 2\cos x \sin 2x + \sin^2 2x) dx \\ &= \int \left(\frac{1}{2} + \frac{1}{2} \cos 2x - 4\cos^2 x \sin x + \frac{1}{2} - \frac{1}{2} \cos 4x \right) dx \\ &= \int \left(1 + \frac{1}{2} \cos 2x - 4\cos^2 x \sin x - \frac{1}{2} \cos 4x \right) dx \\ &= x + \frac{1}{4} \sin 2x + \frac{4}{3} \cos^3 x - \frac{1}{8} \sin 4x + c \end{aligned}$$

$$246) \int \frac{\cos^4 x - \sin^4 x}{\sin^4 2x} dx$$

$$\begin{aligned} &= \int \frac{\cos^4 x - \sin^4 x}{16 \sin^4 x \cos^4 x} dx = \frac{1}{16} \int \left(\frac{1}{\sin^4 x} - \frac{1}{\cos^4 x} \right) dx \\ &= \frac{1}{16} \int (\csc^4 x - \sec^4 x) dx \\ &= \frac{1}{16} \int (\csc^2 x (\cot^2 x + 1) - \sec^2 x (\tan^2 x + 1)) dx \\ &= \frac{1}{16} \int (\cot^2 x \csc^2 x + \csc^2 x - \tan^2 x \sec^2 x - \sec^2 x) dx \\ &= \frac{1}{16} \left[-\frac{1}{3} \cot^3 x - \cot x - \frac{1}{3} \tan^3 x - \tan x \right] + c \end{aligned}$$

$$247) \int \frac{\sin 2x}{\cos^2 2x + 2\cos 2x + 1} dx$$

$$\begin{aligned} &= \int \frac{\sin 2x}{(\cos 2x + 1)^2} dx = \int (\cos 2x + 1)^{-2} \sin 2x dx \\ &= \frac{-1}{2} \frac{(\cos 2x + 1)^{-1}}{-1} + c = \frac{1}{2(\cos 2x + 1)} + c \end{aligned}$$

$$248) \int \cos^3 x \sin^2 x dx$$

$$\begin{aligned} &= \int \cos^2 x \cos x \sin^2 x dx = \int (1 - \sin^2 x) \sin^2 x \cos x dx \\ &= \int (\sin^2 x \cos x - \sin^4 x \cos x) dx \\ &= \frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + c \end{aligned}$$

$$249) \int \frac{\tan x}{1+\cos 2x} dx$$

$$\begin{aligned} &= \int \frac{\tan x}{1+2\cos^2 x - 1} dx = \int \frac{\tan x}{2\cos^2 x} dx = \frac{1}{2} \int \tan x \sec^2 x dx \\ &= \frac{1}{4} \tan^2 x + c \end{aligned}$$

$$250) \int \sin 4x \cos 2x dx$$

$$\begin{aligned} &= \int 2 \sin 2x \cos 2x \cos 2x dx = \int \cos^2 2x \sin 2x dx \\ &= -\frac{1}{3} \cos^3 2x + c \end{aligned}$$

$$251) \int \cos 4x \sqrt{2 - \sin 4x} dx$$

$$\begin{aligned} &= \int (2 - \sin 4x)^{\frac{1}{2}} \cos 4x dx = -\frac{1}{4} \cdot \frac{2}{3} (2 - \sin 4x)^{\frac{3}{2}} + c \\ &= -\frac{1}{6} \sqrt{(2 - \sin 4x)^3} + c \end{aligned}$$

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$$252) \int \sin x \cos x (\sin x + \cos x) dx$$

$$= \int (\sin^2 x \cos x + \cos^2 x \sin x) dx$$

$$= \frac{1}{3} \sin^3 x - \frac{1}{3} \cos^3 x + c$$

$$253) \int \frac{dx}{\cos x \cot x \sqrt{\sec x + 1}}$$

$$= \int \frac{1}{\sqrt{\sec x + 1}} \cdot \frac{1}{\cos x \cot x} dx = \int (\sec x + 1)^{-\frac{1}{2}} \sec x \tan x dx$$

$$= 2(\sec x + 1)^{\frac{1}{2}} + c = 2\sqrt{\sec x + 1} + c$$

$$254) \int \frac{dx}{\sin^2 x - \cos^2 x}$$

$$= \int -\frac{dx}{\cos^2 x - \sin^2 x} dx = - \int \frac{dx}{\cos 2x} = - \int \sec 2x dx$$

$$= -\frac{1}{2} \ln |\sec 2x + \tan 2x| + c$$

$$255) \int \frac{x^2 - 8x + 15}{x^4 - 3x^3} dx$$

$$= \int \frac{(x-3)(x-5)}{x^3(x-3)} dx = \int \frac{(x-5)}{x^3} dx = \int (x^{-2} - 5x^{-3}) dx$$

$$= -\frac{1}{x} + \frac{5}{2x^2} + c$$

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$$\begin{aligned}
 256) \quad & \int \frac{x^3 - 8}{(x^2 + 2x + 4)(\frac{1}{2}x^2 - 2x)^4} dx \\
 &= \int \frac{(x-2)(x^2+2x+4)}{(x^2+2x+4)(\frac{1}{2}x^2-2x)^4} dx = \int \left(\frac{1}{2}x^2 - 2x\right)^{-4} (x-2) dx \\
 &= \frac{\left(\frac{1}{2}x^2 - 2x\right)^{-3}}{-3} + c = -\frac{1}{3\left(\frac{1}{2}x^2 - 2x\right)^3} + c
 \end{aligned}$$

$$\begin{aligned}
 257) \quad & \int \frac{x\sqrt{x^2-4}}{x^2-4} dx \\
 &= \int \frac{x\sqrt{x^2-4}}{\sqrt{x^2-4}\sqrt{x^2-4}} dx = \int \frac{x}{\sqrt{x^2-4}} dx = \int (x^2-4)^{-\frac{1}{2}} x dx \\
 &= \frac{1}{2} \cdot 2(x^2-4)^{\frac{1}{2}} + c = \sqrt{x^2-4} + c
 \end{aligned}$$

$$\begin{aligned}
 258) \quad & \int \frac{(x^2-x)^3}{8x^3} dx \\
 &= \int \left(\frac{x^2-x}{2x}\right)^3 dx = \int \left(\frac{x}{2} - \frac{1}{2}\right)^3 dx = 2 \cdot \frac{1}{4} \left(\frac{x}{2} - \frac{1}{2}\right)^4 + c \\
 &= \frac{1}{2} \left(\frac{x}{2} - \frac{1}{2}\right)^4 + c
 \end{aligned}$$

$$\begin{aligned}
 259) \quad & \int \frac{dx}{\sqrt{x}(\sqrt{3}-\sqrt{x})^3} \\
 &= \int (\sqrt{3}-\sqrt{x})^{-3} \cdot \frac{1}{\sqrt{x}} dx = -\frac{2(\sqrt{3}-\sqrt{x})^{-2}}{-2} + c = \frac{1}{(\sqrt{3}-\sqrt{x})^2} + c
 \end{aligned}$$

$$260) \int \frac{e^{2x}}{\sqrt[3]{3-e^{2x}}} dx$$

$$\begin{aligned} &= \int (3 - e^{2x})^{-\frac{1}{3}} e^{2x} dx = -\frac{1}{2} \cdot \frac{3}{2} (3 - e^{2x})^{\frac{2}{3}} + c \\ &= -\frac{3}{4} \sqrt[3]{(3 - e^{2x})^2} + c \end{aligned}$$

$$261) \int \sin x \cos^3 x \tan^2 x \csc^4 x dx$$

$$= \int \sin x \cos^3 \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^4 x} dx = \int \frac{\cos x}{\sin x} dx = \ln|\sin x| + c$$

$$262) \int \frac{dx}{\sqrt{x}(1+\sqrt{x})}$$

$$= 2 \int \frac{1}{(1+\sqrt{x})} \cdot \frac{1}{2\sqrt{x}} dx = 2 \ln|1 + \sqrt{x}| + c$$

$$263) \int \frac{\sin 3x}{5-2\cos 3x} dx$$

$$= \frac{1}{6} \int \frac{6\sin 3x}{5-2\cos 3x} dx = \frac{1}{6} \ln|5 - 2\cos 3x| + c$$

$$264) \int \frac{(\ln x)^2}{x} dx$$

$$= \int (\ln x)^2 \cdot \frac{1}{x} dx = \frac{1}{3} (\ln x)^3 + c$$

$$265) \int \frac{dx}{(2x+3)^2} dx = \frac{1}{2} \frac{(2x+3)^{-1}}{-1} + c = \frac{-1}{4x+6} + c$$

$$266) \int \frac{\sec^2 3x}{\tan 3x} dx$$

$$= \frac{1}{3} \int \frac{3\sec^2 3x}{\tan 3x} dx = \frac{1}{3} \ln|\tan 3x| + c$$

$$267) \int x\pi^{x^2} dx$$

$$= \frac{1}{2} \int 2x\pi^{x^2} dx = \frac{1}{2} \ln\pi^{x^2} + c$$

$$268) \int \frac{1}{(2+3x)^2} dx$$

$$= \int (2+3x)^{-2} dx = \frac{1}{3} \frac{(2+3x)^{-1}}{-1} + c = \frac{-1}{3(2+3x)} + c$$

$$269) \int e^{\tan x^2} (x \sec^2 x^2) dx$$

$$= \frac{1}{2} e^{\tan x^2} + c$$

$$270) \int \cot x dx$$

$$= \int \frac{\cos x}{\sin x} dx = \ln|\sin x| + c$$

$$271) \int (x^2 - x^5)(2x + 1) dx$$

$$= \int (2x^3 + x^2 - 2x^6 - x^5) dx = \frac{x^4}{2} + \frac{x^3}{3} - \frac{2x^7}{7} - \frac{x^6}{6} + c$$

$$272) \int (3 - x^4)^3 x^3 dx$$

$$= -\frac{1}{4} \cdot \frac{1}{4} (3 - x^4)^4 + c = -\frac{1}{16} (3 - x^4)^4 + c$$

$$273) \int \frac{4x^2 - 1}{x + \frac{1}{2}} dx$$

$$= 4 \int \frac{x^2 - \frac{1}{4}}{x + \frac{1}{2}} dx = 4 \int \frac{(x + \frac{1}{2})(x - \frac{1}{2})}{x + \frac{1}{2}} dx = 4 \int (x - \frac{1}{2}) dx$$

$$= 2x^2 - 2x + c$$

$$274) \int \frac{x^3 - 8}{2 - x} dx$$

$$= \int \frac{(x-2)(x^2 + 2x + 4)}{-(x-2)} dx = - \int (x^2 + 2x + 4) dx$$

$$= -\frac{1}{3}x^3 - x^2 - 4x + c$$

$$275) \int \sqrt{x^2 - x + \frac{1}{4}} dx$$

$$= \int \sqrt{\left(x - \frac{1}{2}\right)^2} dx = \pm \int \left(x - \frac{1}{2}\right) dx = \pm \left(\frac{x^2}{2} - \frac{1}{2}x\right) + c$$

$$276) \int \frac{x}{\sqrt{x-1}} dx$$

$$= \int \frac{x-1+1}{\sqrt{x-1}} dx = \int \frac{x-1}{\sqrt{x-1}} dx + \int \frac{1}{\sqrt{x-1}} dx = \int \sqrt{x-1} dx + \int \frac{1}{\sqrt{x-1}} dx$$

$$= \int (x-1)^{\frac{1}{2}} dx + \int (x-1)^{-\frac{1}{2}} dx = \frac{2}{3} \sqrt{(x-1)^3} + 2 \sqrt{x-1} + c$$

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$$277) \int \frac{\cos^3 x - 1}{\cos^2 x} dx$$

$$= \int \left(\frac{\cos^3 x}{\cos^2 x} - \frac{1}{\cos^2 x} \right) dx = \int (\cos x - \sec^2 x) dx = \sin x - \tan x + c$$

$$278) \int \frac{dx}{x\sqrt{\ln x}}$$

$$= \int \frac{dx}{x(\ln x)^{\frac{1}{2}}} = \int (\ln x)^{-\frac{1}{2}} \cdot \frac{1}{x} dx = 2 (\ln x)^{\frac{1}{2}} + c = 2\sqrt{\ln x} + c$$

$$279) \int \frac{2e^{3x}}{e^x} dx$$

$$= \int 2e^{2x} dx = e^{2x} + c$$

$$280) \int \frac{x^2-x+1}{x(x+1)^2} dx$$

$$= \int \frac{x^2+2x+1+3x}{x(x+1)^2} dx = \int \frac{(x+1)^2}{x(x+1)^2} dx - \int \frac{3x}{x(x+1)^2} dx$$

$$= \int \frac{1}{x} dx - 3 \int \frac{1}{(x+1)^2} dx = \int \frac{1}{x} dx - 3 \int (x+1)^{-2} dx$$

$$= \ln|x| - 3 \cdot \frac{(x+1)^{-1}}{-1} + c = \ln|x| + \frac{3}{x+1} + c$$

$$281) \int \frac{x^5 - 1}{x-1} dx$$

$$\begin{aligned} &= \int \frac{(x-1)(x^4 + x^3 + x^2 + x + 1)}{x-1} dx = \int (x^4 + x^3 + x^2 + x + 1) dx \\ &= \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + c \end{aligned}$$

$$282) \int \frac{3x^2 + 2}{x^3 + 2x} dx$$

$$\begin{aligned} &= \int \frac{2x^2 + x^2 + 2}{x(x^2 + 2)} dx = \int \left(\frac{x^2 + 2}{x(x^2 + 2)} + \frac{2x^2}{x(x^2 + 2)} \right) dx \\ &= \int \left(\frac{1}{x} + \frac{2x}{(x^2 + 2)} \right) dx = \ln|x| + \ln|x^2 + 2| + c = \ln|x^3 + 2x| + c \end{aligned}$$

$$OR: \int \frac{3x^2 + 2}{x^3 + 2x} dx = \ln|x^3 + 2x| + c$$

$$283) \int \frac{(x^2 - 4)(x + 3)}{x+2} dx$$

$$\begin{aligned} &= \int \frac{(x+2)(x-2)(x+3)}{x+2} dx = \int (x-2)(x+3) dx \\ &= \int (x^2 + 3x - 2x - 6) dx = \int (x^2 + x - 6) dx \\ &= \frac{x^3}{3} + \frac{x^2}{2} - 6x + c \end{aligned}$$

$$284) \int \frac{x^3 - 8}{x-2} dx$$

$$\begin{aligned} &= \int \frac{(x-2)(x^2 + 2x + 4)}{(x-2)} dx = \int (x^2 + 2x + 4) dx \\ &= \frac{x^3}{3} + x^2 + 4x + c \end{aligned}$$

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$$285) \int \frac{x^3 - x^2}{x-1} dx$$

$$= \int \frac{x^2(x-1)}{x-1} dx = \int x^2 dx = \frac{x^3}{3} + c$$

$$286) \int \frac{(x+2)^2}{x} dx$$

$$= \int \frac{x^2 + 4x + 4}{x} dx = \int \left(x + 4 + \frac{4}{x} \right) dx = \frac{x^2}{2} + 4x + \ln|x| + c$$

$$287) \int \frac{x^6 + x^5 + x^4 + x^3 + 1}{2x^3} dx$$

$$= \int \left(\frac{x^6}{2x^3} + \frac{x^5}{2x^3} + \frac{x^4}{2x^3} + \frac{x^3}{2x^3} + \frac{1}{2x^3} \right) dx$$

$$= \int \left(\frac{x^3}{2} + \frac{x^2}{2} + \frac{x}{2} + \frac{1}{2} + \frac{1}{2}x^{-3} \right) dx$$

$$= \frac{x^4}{8} + \frac{x^3}{6} + \frac{x^2}{4} + \frac{1}{2}x - \frac{1}{4x^2} + c$$

$$288) \int \frac{x^3 + x^2 + x}{x^3 - 1} dx$$

$$= \int \frac{x(x^2 + x + 1)}{(x-1)(x^2 + x + 1)} dx = \int \frac{x}{x-1} dx = \int \frac{x-1+1}{x-1} dx$$

$$= \int \frac{x-1}{x-1} dx + \int \frac{1}{x-1} dx = \int dx + \int \frac{1}{x-1} dx = x + \ln|x-1| + c$$

$$289) \int (x+1)^2 \sqrt{x} dx$$

$$\begin{aligned} &= \int (x^2 + 2x + 1)x^{\frac{1}{2}} dx = \int \left(x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + x^{\frac{1}{2}}\right) dx \\ &= \frac{2}{7}x^{\frac{7}{2}} + 2 \cdot \frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}} + c = \frac{2}{7}\sqrt{x^7} + \frac{4}{5}\sqrt{x^5} + \frac{2}{3}\sqrt{x^3} + c \end{aligned}$$

$$290) \int (x^2 + 1)(x - 1) dx$$

$$= \int (x^3 - x^2 + x - 1) dx = \frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} - x + c$$

$$291) \int \frac{3}{x+2} dx$$

$$= 3 \int \frac{1}{x+2} dx = 3 \ln|x+2| + c = \ln|(x+2)^3| + c$$

$$292) \int \frac{x^3}{\sqrt[3]{x^6+x^8}} dx$$

$$\begin{aligned} &= \int \frac{x^3}{\sqrt[3]{x^6(1+x^2)}} dx = \int \frac{x^3}{x^2 \sqrt[3]{1+x^2}} dx = \int \frac{x}{\sqrt[3]{1+x^2}} dx \\ &= \int (1+x^2)^{-\frac{1}{3}} x dx = \frac{1}{2} \cdot \frac{3}{2} (1+x^2)^{\frac{2}{3}} + c = \frac{3}{4} \sqrt[3]{(1+x^2)^2} + c \end{aligned}$$

$$293) \int \sqrt{x^4 + 2x^2 + 1} x dx$$

$$\begin{aligned} &= \int \sqrt{(x^2 + 1)^2} x dx = \pm \int (x^2 + 1) x dx = \pm \frac{1}{2} \cdot \frac{1}{2} (x^2 + 1)^2 + c \\ &= \pm \frac{1}{4} (x^2 + 1)^2 + c \end{aligned}$$

$$294) \int \sqrt{x^2 + x^4} dx$$

$$\begin{aligned} &= \int \sqrt{x^2(1+x^2)} dx = \pm \int \sqrt{1+x^2} x dx = \pm \int (1+x^2)^{\frac{1}{2}} x dx \\ &= \pm \frac{1}{2} \cdot \frac{2}{3} (1+x^2)^{\frac{3}{2}} + c = \pm \frac{1}{3} \sqrt{(1+x^2)^3} + c \end{aligned}$$

$$295) \int (\sqrt{x^3} + \sqrt{x})^2 dx$$

$$\begin{aligned} &= \int (x^3 + 2\sqrt{x^4} + x) dx = \int (x^3 \pm 2x^2 + x) dx \\ &= \frac{x^4}{4} \pm \frac{2}{3}x^3 + \frac{x^2}{2} + c \end{aligned}$$

$$296) \int \cos 6x \sin 3x dx$$

$$\begin{aligned} &= \int (2\cos^2 3x - 1) \sin 3x dx = \int (2\cos^2 3x \sin 3x - \sin 3x) dx \\ &= -\frac{2}{3} \cdot \frac{1}{3} \cos^3 3x + \frac{1}{3} \cos 3x + c = -\frac{2}{9} \cos^3 3x + \frac{1}{3} \cos 3x + c \end{aligned}$$

$$297) \int \cos 3x \cos \frac{3x}{2} dx$$

$$\begin{aligned} &= \int \left(1 - 2\sin^2 \frac{3x}{2}\right) \cos \frac{3x}{2} dx = \int \left(\cos \frac{3x}{2} - 2\sin^2 \frac{3x}{2} \cos \frac{3x}{2}\right) dx \\ &= \frac{2}{3} \sin \frac{3x}{2} - \frac{4}{9} \sin^3 \frac{3x}{2} + c \end{aligned}$$

$$298) \int \frac{2-x}{(x-1)^2} dx$$

$$\begin{aligned} &= \int \frac{1+1-x}{(x-1)^2} dx = \int \frac{x+1}{(x-1)^2} dx - \int \frac{1}{(x-1)^2} dx \\ &= \int \frac{1}{x-1} dx - \int (x-1)^{-2} dx = \ln|x-1| - \frac{(x-1)^{-1}}{-1} + c \\ &= \ln|x-1| + \frac{1}{x-1} + c \end{aligned}$$

$$299) \int \cos^2 \frac{x}{3} \sin^2 \frac{x}{3} dx$$

$$\begin{aligned} &= \frac{1}{4} \int \cos^2 \frac{x}{3} \sin^2 \frac{x}{3} dx = \frac{1}{4} \int \sin^2 \frac{2x}{3} dx = \frac{1}{4} \int \left(\frac{1}{2} - \frac{1}{2} \cos \frac{4x}{3} \right) dx \\ &= \frac{1}{8} \left(x - \frac{1}{2} \cdot \frac{3}{4} \sin \frac{4x}{3} \right) + c = \frac{x}{8} - \frac{3}{64} \sin \frac{4x}{3} + c \end{aligned}$$

$$300) \int \frac{x^3}{(x+1)^5} dx$$

$$= \int \left(\frac{x}{x+1} \right)^3 \frac{1}{(x+1)^2} dx = \frac{1}{4} \left(\frac{x}{x+1} \right)^4 + c$$

$$301) \int \frac{2x-6}{\sqrt{3-x}} dx$$

$$\begin{aligned} &= -2 \int \frac{3-x}{\sqrt{3-x}} dx = -2 \int \frac{\sqrt{3-x} \sqrt{3-x}}{\sqrt{3-x}} dx = -2 \int \sqrt{3-x} dx \\ &= -2 \int (3-x)^{\frac{1}{2}} dx = 2 \cdot \frac{2}{3} (3-x)^{\frac{3}{2}} + c = \frac{4}{3} \sqrt{(3-x)^3} + c \end{aligned}$$

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$$302) \int \frac{(x^2-x)^{10}}{x^{10}} dx$$

$$\begin{aligned} &= \int \frac{x(x-1)]^{10}}{x^{10}} dx = \int \frac{x^{10} (x-1)^{10}}{x^{10}} dx = \int (x-1)^{10} dx \\ &= \frac{(x-1)^{11}}{11} + x \end{aligned}$$

$$303) \int \frac{x-5\sqrt{x}+6}{x-3\sqrt{x}} dx$$

$$= \int \frac{(\sqrt{x}-3)(\sqrt{x}-2)}{\sqrt{x}(\sqrt{x}-3)} dx = \int \frac{\sqrt{x}-2}{\sqrt{x}} dx = (\sqrt{x} - 2)^2 + c$$

$$304) \int \sqrt{x^2 - 2x^4} dx$$

$$\begin{aligned} &= \pm \int \sqrt{1 - 2x^2} x dx = \pm \int (1 - 2x^2)^{\frac{1}{2}} x dx \\ &= \pm \left[-\frac{1}{4} \cdot \frac{2}{3} (1 - 2x^2)^{\frac{3}{2}} \right] + c = \pm \left[-\frac{1}{6} \sqrt{(1 - 2x^2)^3} \right] + c \end{aligned}$$

$$305) \int \frac{(1+x)^2}{\sqrt{x}} dx$$

$$\begin{aligned} &= \int \frac{1+2x+x^2}{\sqrt{x}} dx = \int \left(x^{-\frac{1}{2}} + 2x^{\frac{1}{2}} + x^{\frac{3}{2}} \right) dx \\ &= 2x^{\frac{1}{2}} + \frac{4}{3}x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + c = 2\sqrt{x} + \frac{4}{3}\sqrt{x^3} + \frac{2}{5}\sqrt{x^5} + c \end{aligned}$$

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$$306) \int \frac{(e^{-x} + e^{3x})^2}{e^x} dx$$

$$= \int \frac{e^{-2x} + 2e^{2x} + e^{6x}}{e^x} dx = \int (e^{-3x} + 2e^x + e^{5x}) dx$$

$$= -\frac{1}{3}e^{-3x} + 2e^x + \frac{1}{5}e^{5x} + c$$

$$307) \int_2^3 \frac{2}{x^2-1} dx$$

$$= \int_2^3 \frac{x-x+1+1}{x^2-1} dx = \int_2^3 \frac{x+1}{(x-1)(x+1)} dx - \int_2^3 \frac{x-1}{(x-1)(x+1)} dx$$

$$= \int_2^3 \frac{1}{x-1} dx - \int_2^3 \frac{1}{x+1} dx = [\ln|x-1|]_2^3 - [\ln|x+1|]_2^3$$

$$= [\ln 2 - \ln 1] - [\ln 4 - \ln 3] = \ln 2 + \ln 3 - \ln 4$$

$$= \ln 3 - \ln 2 = \ln \frac{3}{2}$$

$$308) \int \sqrt{2 - 2\cos 2x} dx$$

$$= \int \sqrt{2 - 2(1 - 2\sin^2 x)} dx = \int \sqrt{2 - 2 + 4\sin^2 x} dx$$

$$= \int \sqrt{4\sin^2 x} dx = \pm 2 \int \sin x dx = \pm 2\cos x + c$$

$$309) \int \frac{\ln x}{x} dx$$

$$= \int \ln x \cdot \frac{1}{x} dx = \frac{(\ln x)^2}{2} + c$$

$$310) \int_0^{\ln 2} e^{-x} dx$$

$$= [-e^{-x}]_0^{\ln 2} = -(e^{-\ln 2} - e^0) = -(e^{\ln 2^{-1}} - 1) = -\left(\frac{1}{2} - 1\right) = \frac{1}{2}$$

$$311) \int_0^1 (1 + e^x)^2 e^x dx$$

$$= \frac{1}{3} [(1 + e^x)^3]_0^1 = \frac{1}{3} [(1 + e^1)^3 - (1 + e^0)^3] = \frac{1}{3} ((1 + e)^3 - 8)$$

$$312) \int_0^1 \frac{3x^2+4}{x^3+4x+1} dx$$

$$= [\ln|x^3 + 4x + 1|]_0^1 = \ln|1 + 4 + 1| - \ln 1 = \ln 6 - 0 = \ln 6$$

$$313) \int_1^4 \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx$$

$$= \int_1^4 e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} dx = [e^{\sqrt{x}}]_1^4 = e^{\sqrt{4}} - e^{\sqrt{1}} = e^2 - e$$

$$314) \int_0^{\frac{\pi}{2}} e^{\cos x} \sin x dx$$

$$= -[e^{\cos x}]_0^{\frac{\pi}{2}} = -\left[e^{\cos \frac{\pi}{2}} - e^{\cos 0}\right] = -(e^0 - e^1) = e - 1$$

$$315) \int_0^4 \frac{x}{x^2+9} dx$$

$$= \frac{1}{2} [\ln|x^2 + 9|]_0^4 = \frac{1}{2} [\ln|16 + 9| - \ln|9|] = \frac{1}{2} [\ln 25 - \ln 9]$$

$$= \frac{1}{2} \ln \frac{25}{9} = \ln \frac{5}{3}$$

$$316) \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2 x}{2+\tan x} dx$$

$$= [\ln|2 + \tan x|]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \left(\ln\left(2 + \tan\frac{\pi}{4}\right) \right) - \left(\ln\left(2 - \tan\frac{\pi}{4}\right) \right)$$

$$= \ln(2 + 1) - \ln(2 - 1) = \ln 3 - \ln 1 = \ln 3$$

$$317) \int_{\ln 3}^{\ln 5} e^{2x} dx$$

$$= \frac{1}{2} [e^{2x}]_{\ln 3}^{\ln 5} = \frac{1}{2} [e^{2\ln 5} - e^{2\ln 3}] = \frac{1}{2} [e^{\ln 5^2} - e^{\ln 3^2}]$$

$$= \frac{1}{2} (25 - 9) = \frac{1}{2} (16) = 8$$

$$318) \int \sec^2 3x e^{\tan 3x} dx$$

$$= \frac{1}{3} e^{\tan 3x} + c$$

$$319) \int \frac{\sqrt{x^4+x^6}}{x} dx$$

$$= \pm \int \frac{x^2 \sqrt{1+x^2}}{x} dx = \pm \int (1+x^2)^{\frac{1}{2}} x dx = \pm \frac{1}{2} \cdot \frac{2}{3} (1+x^2)^{\frac{3}{2}} + c$$

$$= \pm \frac{1}{3} \sqrt{(1+x^2)^3} + c$$

$$320) \int \frac{dx}{\sqrt{9-6x+x^2}}$$

$$= \int \frac{dx}{\sqrt{(x-3)^2}} = \pm \int \frac{dx}{x-3} = \pm \ln|x-3| + c$$

$$321) \int \frac{dx}{\sqrt{x+1}}$$

$$= \int \frac{dx}{(x+1)^{\frac{1}{2}}} = \int (x+1)^{-\frac{1}{2}} dx = 2(x+1)^{\frac{1}{2}} + c = 2\sqrt{x+1} + c$$

$$322) \int \frac{dx}{x^2+2x+1}$$

$$= \int \frac{dx}{(x+1)^2} = \int (x+1)^{-2} dx = \frac{(x+1)^{-1}}{-1} + c = -\frac{1}{x+1} + c$$

$$323) \int \left(x^2 - x + \frac{1}{4}\right)^2 dx$$

$$= \int \left(\left(x - \frac{1}{2}\right)^2\right)^2 dx = \int \left(x - \frac{1}{2}\right)^4 dx = \frac{1}{5} \left(x - \frac{1}{2}\right)^5 + c$$

$$324) \int (3y+1)^5 dy$$

$$= \frac{1}{3} \cdot \frac{1}{6} (3y+1)^6 + c = \frac{1}{18} (3y+1)^6 + c$$

$$325) \int \sqrt{\frac{\sqrt{x}+1}{x}} dx$$

$$= \int \frac{\sqrt{\sqrt{x}+1}}{\sqrt{x}} dx = \int \frac{\left(x^{\frac{1}{2}}+1\right)^{\frac{1}{2}}}{x^{\frac{1}{2}}} dx = \left(x^{\frac{1}{2}}+1\right)^{\frac{1}{2}} x^{-\frac{1}{2}} dx$$

$$= 2 \cdot \frac{2}{3} \left(x^{\frac{1}{2}}+1\right)^{\frac{3}{2}} + c = \frac{4}{3} \sqrt{\left(\sqrt{x}+1\right)^3} + c$$

$$326) \int (x+2)^{\frac{3}{2}} dx$$

$$= \frac{2}{5} (x+2)^{\frac{5}{2}} + c = \frac{2}{5} \sqrt{(x+2)^5} + c$$

$$327) \int (\sqrt[3]{x} + 2)^2 dx$$

$$= \int (\sqrt[3]{x^2} + 4\sqrt[3]{x} + 4) dx = \int \left(x^{\frac{2}{3}} + 4x^{\frac{1}{3}} + 4 \right) dx$$

$$= \frac{3}{5} x^{\frac{5}{3}} + 4 \cdot \frac{3}{4} x^{\frac{4}{3}} + 4x + c = \frac{3}{5} \sqrt[3]{x^5} + 3\sqrt[3]{x^4} + 4x + c$$

$$328) \int \left(\frac{1}{x} + 1 \right)^2 x^2 dx$$

$$= \int \left(\frac{1}{x^2} + \frac{2}{x} + 1 \right) x^2 dx = \int (1 + 2x + x^2) dx$$

$$= x + x^2 + \frac{x^3}{3} + c$$

$$329) \int \sec^2 3x \csc^2 3x dx$$

$$= \int (\tan^2 3x + 1) \csc^2 3x dx = \int (\tan^2 3x \csc^2 3x + \csc^2 3x) dx$$

$$= \int \left(\frac{\sin^2 3x}{\cos^2 3x} \frac{1}{\sin^2 3x} + \csc^2 3x \right) dx = \int (\sec^2 3x + \csc^2 3x) dx$$

$$= \frac{1}{3} \tan 3x - \frac{1}{3} \cot 3x + c$$

$$OR: \int \sec^2 3x \csc^2 3x dx = \int \frac{1}{\sin^2 3x \cos^2 3x} dx$$

$$\begin{aligned}
 &= \int \frac{4}{4\sin^2 3x \cos^2 3x} dx = 4 \int \frac{1}{(2\sin 3x \cos 3x)^2} dx \\
 &= 4 \int \frac{1}{\sin^2 6x} dx = 4 \int \csc^2 6x dx = -\frac{2}{3} \cot 6x + c \\
 \text{OR: } &\int \sec^2 3x \csc^2 3x dx = \int \frac{1}{\sin^2 3x \cos^2 3x} dx \\
 &= \int \frac{\sin^2 3x + \cos^2 3x}{\sin^2 3x \cos^2 3x} dx = \int (\sec^2 3x + \csc^2 3x) \\
 &= \frac{1}{3} \tan 3x - \frac{1}{3} \cot 3x + c
 \end{aligned}$$

330) $\int_3^8 \frac{x}{\sqrt{x+1}} dx$

$$\begin{aligned}
 &= \int_3^8 \frac{x+1-1}{\sqrt{x+1}} dx = \int_3^8 \frac{x+1}{\sqrt{x+1}} dx - \int_3^8 \frac{1}{\sqrt{x+1}} dx \\
 &= \int_3^8 \frac{\sqrt{x+1} \cdot \sqrt{x+1}}{\sqrt{x+1}} dx - \int_3^8 \frac{1}{\sqrt{x+1}} dx = \int_3^8 \sqrt{x+1} dx - \int_3^8 \frac{1}{\sqrt{x+1}} dx \\
 &= \int_3^8 (x+1)^{\frac{1}{2}} dx - \int_3^8 (x+1)^{-\frac{1}{2}} dx = \frac{2}{3} \left[(x+1)^{\frac{3}{2}} \right]_3^8 - 2 \left[(x+1)^{\frac{1}{2}} \right]_3^8 \\
 &= \frac{2}{3} \left[\sqrt{(x+1)^3} \right]_3^8 - 2 \left[\sqrt{x+1} \right]_3^8 = \frac{2}{3} [27 - 8] - 2[3 - 2] = \frac{38}{3} - 2 = \frac{32}{3}
 \end{aligned}$$

331) $\int \frac{\sqrt{x^3} - \sqrt[3]{x}}{6\sqrt[4]{x}} dx$

$$\begin{aligned}
 &= \frac{1}{6} \int \left(\frac{\sqrt{x^3}}{\sqrt[4]{x}} - \frac{\sqrt[3]{x}}{\sqrt[4]{x}} \right) dx = \frac{1}{6} \int \left(x^{\frac{3}{2}} x^{-\frac{1}{4}} - x^{\frac{1}{3}} x^{-\frac{1}{4}} \right) dx \\
 &= \frac{1}{6} \int \left(x^{\frac{5}{4}} - x^{\frac{1}{12}} \right) dx = \frac{1}{6} \left[\frac{4}{9} x^{\frac{9}{4}} - \frac{12}{13} x^{\frac{13}{12}} \right] + c = \frac{2}{27} x^{\frac{9}{4}} - \frac{2}{13} x^{\frac{13}{12}} + c \\
 &= \frac{2}{27} \sqrt[4]{x^9} - \frac{2}{13} \sqrt[12]{x^{13}} + c
 \end{aligned}$$

$$332) \int \frac{(3x^2-4)^2 - 16}{x^2} dx$$

$$\begin{aligned} &= \int \frac{[(3x^2-4)-4][(3x^2-4)+4]}{x^2} dx = \int \frac{(3x^2-8)3x^2}{x^2} dx = \int (9x^2 - 24) dx \\ &= \frac{9x^3}{3} - 24x + c = 3x^3 - 24x + c \end{aligned}$$

$$333) \int \frac{x^3+27}{x+3} dx$$

$$= \int \frac{(x+3)(x^2-3x+9)}{x+3} dx = \int (x^2 - 3x + 9) dx = \frac{x^3}{3} - \frac{3x^2}{2} + 9x + c$$

$$334) \int \frac{(x^4-16)}{x+2} dx$$

$$\begin{aligned} &= \int \frac{(x^2+4)(x^2-4)}{x+2} dx = \int \frac{(x^2+4)(x-2)(x+2)}{x+2} dx = \int (x^3 - 2x^2 + 4x - 8) dx \\ &= \frac{x^4}{4} - \frac{2x^3}{3} + \frac{4x^2}{2} - 8x + c = \frac{1}{4}x^4 - \frac{2}{3}x^3 + 2x^2 - 8x + c \end{aligned}$$

$$335) \int \frac{dx}{(x^2+8x+16)^3}$$

$$\begin{aligned} &= \int \frac{dx}{((x+4)^2)^3} = \int \frac{dx}{(x+4)^6} = \int (x+4)^{-6} dx \\ &= \frac{(x+4)^{-5}}{-5} + c = -\frac{1}{(x+4)^5} + c \end{aligned}$$

$$336) \int_0^8 \sqrt{x^2 - 14x + 49} dx$$

$$= \int_0^8 \sqrt{(x-7)^2} dx = \int_0^7 (7-x) dx + \int_7^8 (x-7) dx$$

$$= \left[7x - \frac{x^2}{2} \right]_0^7 + \left[\frac{x^2}{2} - 7x \right]_7^8$$

$$= [(49 - 49/2) - (0)] + \left[(32 - 56) - \left(\frac{49}{2} - 49 \right) \right] = 25$$

$$337) \int_0^3 \sqrt[3]{(3x-1)^2} dx$$

$$= \int_0^3 ((3x-1)^2)^{\frac{1}{3}} dx = \int_0^3 (3x-1)^{\frac{2}{3}} dx = \frac{1}{3} \left[\frac{(3x-1)^{\frac{5}{3}}}{\frac{5}{3}} \right]_0^3$$

$$= \frac{1}{3} \cdot \frac{3}{5} \left[(3x-1)^{\frac{5}{3}} \right]_0^3 = \frac{1}{5} \left(\left[(9-1)^{\frac{5}{3}} \right] - \left[(0-1)^{\frac{5}{3}} \right] \right) = \frac{1}{5} (32+1) = \frac{33}{5}$$

$$338) \int \frac{7}{x^2-3x-10} dx$$

$$= \int \frac{x+2+5-x}{(x-5)(x+2)} dx = \int \frac{x+2}{(x-5)(x+2)} dx - \int \frac{x-5}{(x-5)(x+2)} dx$$

$$= \int \frac{1}{x-5} dx - \int \frac{1}{x+2} dx = \ln|x-5| - \ln|x+2| + c = \ln \left| \frac{x-5}{x+2} \right| + c$$

$$339) \int x \sqrt{1-2x^2} dx$$

$$= \int (1-2x^2)^{\frac{1}{2}} x dx = -\frac{1}{4} \cdot \frac{2}{3} (1-2x^2)^{\frac{3}{2}} + c$$

$$= -\frac{1}{6} \sqrt{(1-2x^2)^3} + c$$

$$340) \int \sin^7 x \cos x dx$$

$$= \frac{\sin^8 x}{8} + c$$

$$341) \int 3^{x^2-1} x dx$$

$$= \frac{3^{x^2-1}}{2 \ln 3} + c$$

$$342) \int (x^5 + 4y) dx$$

$$= \frac{x^6}{6} + 4yx + c$$

$$343) \int e^{\cot x + 1} \csc^2 x dx$$

$$= -e^{\cot x + 1} + c$$

$$344) \int e^{x^2-1} x dx$$

$$= \frac{1}{2} e^{x^2-1} + c$$

$$345) \int \frac{x^2 - 2x + 1}{x} dx$$

$$= \int \left(\frac{x^2}{x} - \frac{2x}{x} + \frac{1}{x} \right) dx = \int \left(x - 2 + \frac{1}{x} \right) dx$$

$$= x^2 - 2x + \ln|x| + c$$

$$346) \int \frac{e^x}{1+e^x} dx$$

$$= \ln|1 + e^x| + c$$

$$347) \int \frac{x^2}{1-2x^3} dx$$

$$= -\frac{1}{6} \int \frac{-6x^2}{1-2x^3} = -\frac{1}{6} \ln|1 - 2x^3| + c$$

$$348) \int \frac{\sec x \tan x}{\sqrt{\sec x}} dx$$

$$= \int \frac{\sqrt{\sec x} \sqrt{\sec x} \tan x}{\sqrt{\sec x}} dx = \int \sqrt{\sec x} \tan x dx = \int \sec^{\frac{1}{2}} x \tan x dx$$

$$= \int \sec^{-\frac{1}{2}} x \sec x \tan x dx = 2 \sec^{\frac{1}{2}} x + c = 2\sqrt{\sec x} + c$$

$$349) \int \frac{7-\ln x}{x(3+\ln x)} dx$$

$$= \int \left(\frac{7}{x(3+\ln x)} - \frac{\ln x}{x(3+\ln x)} \right) dx = 7 \int \left(\frac{\frac{1}{x}}{x(3+\ln x)} \right) dx - \int \frac{\ln x + 3 - 3}{x(3+\ln x)} dx$$

$$= 7 \int \left(\frac{\frac{1}{x}}{(3+\ln x)} \right) dx - \int \frac{1}{x} dx + 3 \int \frac{\frac{1}{x}}{(3+\ln x)} dx$$

$$= 7 \ln|3 + \ln x| - \ln|x| + 3 \ln|3 + \ln x| + c$$

$$= 10 \ln|3 + \ln x| - \ln|x| + c = \ln|(3 + \ln x)^{10}| - \ln|x| + c$$

$$= \ln \left| \frac{(3+\ln x)^{10}}{x} \right| + c$$

$$350) \int \frac{7x^2+3x+1}{2x} dx$$

$$\begin{aligned} &= \frac{1}{2} \int \left(\frac{7x^2}{x} + \frac{3x}{x} + \frac{1}{x} \right) dx = \frac{1}{2} \int \left(7x + 3 + \frac{1}{x} \right) dx \\ &= \frac{1}{2} \left[\frac{7x^2}{2} + 3x + \ln|x| \right] + c = \frac{7x^2}{4} + \frac{3x}{2} + \frac{1}{2} \ln|x| + c \end{aligned}$$

$$351) \int \frac{(x^2-4)^2}{2x} dx$$

$$\begin{aligned} &= \frac{1}{2} \int \frac{x^4 - 8x^2 + 16}{x} dx = \frac{1}{2} \int \left(\frac{x^4}{x} - \frac{8x^2}{x} + \frac{16}{x} \right) dx \\ &= \frac{1}{2} \int \left(x^3 - 8x + \frac{16}{x} \right) dx = \frac{1}{2} \left[\frac{x^4}{4} - \frac{8}{2} x^2 + 16 \ln|x| \right] + c \\ &= \frac{x^4}{8} - 2x^2 + 8 \ln|x| + c \end{aligned}$$

$$352) \int_{-1}^0 \frac{1}{4-5x} dx$$

$$= -\frac{1}{5} [\ln|4-5x|]_{-1}^0 = -\frac{1}{5} [\ln 4 - \ln 9] = -\frac{1}{5} \ln \frac{4}{9} = \ln \sqrt[5]{\frac{9}{4}}$$

$$353) \int \frac{x^3+3x}{(x^2+3)^4} dx$$

$$\begin{aligned} &= \int \frac{x(x^2+3)}{(x^2+3)^4} dx = \int (x^2+3)^{-4} x dx = \frac{-1}{2} \cdot \frac{1}{2} (x^2+3)^{-3} + c \\ &= \frac{-1}{4(x^2+3)^3} + c \end{aligned}$$

$$354) \int_4^8 x\sqrt{x^2 - 15} dx$$

$$\begin{aligned} &= \int_4^8 (x^2 - 15)^{\frac{1}{2}} x dx = \frac{1}{2} \cdot \frac{2}{3} \left[(x^2 - 15)^{\frac{3}{2}} \right]_4^8 = \frac{1}{3} \left[\sqrt{(x^2 - 15)^3} \right]_4^8 \\ &= \frac{1}{3} \left[\sqrt{(64 - 15)^3} - \sqrt{(16 - 15)^3} \right] = \frac{1}{3} ((7)^3 - 1) = \frac{1}{3} (342) = 114 \end{aligned}$$

$$355) \int_0^3 \sqrt[3]{1 - 3x} dx$$

$$\begin{aligned} &= \int_0^3 (1 - 3x)^{\frac{1}{3}} dx = -\frac{1}{3} \cdot \frac{3}{4} \left[(1 - 3x)^{\frac{4}{3}} \right]_0^3 = -\frac{1}{4} \left[\sqrt[3]{(1 - 3x)^4} \right]_0^3 \\ &= -\frac{1}{4} \left[\sqrt[3]{(1 - 9)^4} - \sqrt[3]{(1 - 0)^4} \right] = -\frac{1}{4} \left[\sqrt[3]{(-8)^4} - \sqrt[3]{1} \right] \\ &= -\frac{1}{4} [(-2)^4 - 1] = -\frac{1}{4} (16 - 1) = -\frac{15}{4} \end{aligned}$$

$$356) \int_0^1 x\sqrt{1 - x} dx$$

$$\begin{aligned} &= \int_0^1 (x + 1 - 1)\sqrt{1 - x} dx = \int_0^1 [(x - 1)\sqrt{1 - x} + \sqrt{1 - x}] dx \\ &= \int_0^1 \left[-(1 - x)(1 - x)^{\frac{1}{2}} + (1 - x)^{\frac{1}{2}} \right] dx \\ &= \int_0^1 \left[-(1 - x)^{\frac{3}{2}} + (1 - x)^{\frac{1}{2}} \right] dx = \left[\frac{2}{5} (1 - x)^{\frac{5}{2}} - \frac{2}{3} (1 - x)^{\frac{3}{2}} \right]_0^1 \\ &= \left[(0) - \left(\frac{2}{5} - \frac{2}{3} \right) \right] = -\frac{2}{5} + \frac{2}{3} = \frac{-6+10}{15} = \frac{4}{15} \end{aligned}$$

$$357) \int \frac{2x^2 - 5x - 7}{x-3} dx$$

$$\begin{aligned} &= \int \frac{(2x-7)(x+1)}{x-3} dx = \int \frac{(2x-7)(x-3+4)}{x-3} dx \\ &= \int \frac{(2x-7)(x-3)}{x-3} dx + 4 \int \frac{2x-7}{x-3} dx = \int (2x-7)dx + 4 \int \frac{2x-6-1}{x-3} dx \\ &= \int (2x-7)dx + 4 \int \frac{2(x-3)}{x-3} dx - 4 \int \frac{1}{x-3} dx \\ &= \int (2x-7)dx + 8 \int dx - 4 \int \frac{1}{x-3} dx \\ &= x^2 - 7x + 8x - 4 \ln|x-3| + c = x^2 + x - 4 \ln|x-3| + c \end{aligned}$$

$$358) \int_{-3}^3 \frac{x^2 + 2x}{(x+1)^2} dx$$

$$\begin{aligned} &= \int_{-3}^3 \frac{x^2 + 2x + 1 - 1}{(x+1)^2} dx = \int_{-3}^3 \frac{x^2 + 2x + 1}{(x+1)^2} dx - \int_{-3}^3 \frac{1}{(x+1)^2} dx \\ &= \int_{-3}^3 \frac{(x+1)^2}{(x+1)^2} dx - \int_{-3}^3 \frac{1}{(x+1)^2} dx = \int_{-3}^3 dx - \int_{-3}^3 (x+1)^{-2} dx \\ &= [x]_{-3}^3 - \left[\frac{(x+1)^{-1}}{-1} \right]_{-3}^3 = [x]_{-3}^3 + \left[\frac{1}{x+1} \right]_{-3}^3 \\ &= [3 + 3] + \left[\frac{1}{4} + \frac{1}{2} \right] = 6 + \frac{1}{4} + \frac{1}{2} = \frac{27}{4} \end{aligned}$$

$$359) \int_0^6 |3-x| dx$$

$$3-x=0 \Rightarrow x=3 \in [0, 6]$$

$$=\int_0^3 (3-x)dx + \int_3^6 (x-3)dx$$

$$=\left[3x - \frac{x^2}{2}\right]_0^3 + \left[\frac{x^2}{2} - 3x\right]_3^6 = \left(9 - \frac{9}{2}\right) + \left((18 - 18) - \left(\frac{9}{2} - 9\right)\right)$$

$$=\frac{18-9}{2} + \left[-\left(\frac{9-18}{2}\right)\right] = \frac{9}{2} + \frac{9}{2} = \frac{18}{2} = 9$$

$$360) \int \frac{x-8}{\sqrt[3]{x^2+2\sqrt[3]{x+4}}} dx$$

$$=\int \frac{x-8}{\sqrt[3]{x^2+2\sqrt[3]{x+4}}} \frac{\sqrt[3]{x}-2}{\sqrt[3]{x}-2} dx = \int \frac{(x-8)(\sqrt[3]{x}-2)}{x-8} dx = \int (\sqrt[3]{x}-2) dx$$

$$=\int \left(x^{\frac{1}{3}} - 2\right) dx = \frac{3}{4} x^{\frac{4}{3}} - 2x + c = \frac{3}{4} \sqrt[3]{x^4} - 2x + c$$

$$361) \int (\sec x \tan x)^4 dx$$

$$=\int \sec^4 x \tan^4 x dx = \int \tan^4 x (\tan^2 x + 1) \sec^2 x dx$$

$$=\int (\tan^6 x \sec^2 x + \tan^4 x \sec^2 x) = \frac{1}{7} \tan^7 x + \frac{1}{5} \tan^5 x + c$$

$$362) \int \frac{\cot x}{\sin x} dx$$

$$=\int \csc x \cot x dx = -\csc x + c$$

$$363) \int \frac{\sec^6 x}{\cot x} dx$$

$$= \int \sec^6 x \tan x dx = \int \sec^5 x \sec x \tan x dx = \frac{1}{6} \sec^6 x + c$$

$$364) \int \cos^4 x \sin 2x dx$$

$$= \int \cos^4 x (2 \sin x \cos x) dx = 2 \int \cos^5 x \sin x dx = -2 \frac{\cos^6 x}{6} + c$$

$$= -\frac{1}{3} \cos^6 x + c$$

$$365) \int \frac{x dx}{(x^2 + 4x + 4)^5}$$

$$= \int \frac{x dx}{[(x+2)^2]^5} = \int \frac{x+2-2}{(x+2)^{10}} dx = \int \frac{x+2}{(x+2)^{10}} dx - \int \frac{2}{(x+2)^{10}} dx$$

$$= \int \frac{1}{(x+2)^9} dx - 2 \int \frac{1}{(x+2)^{10}} dx = \int (x+2)^{-9} dx - 2 \int (x+2)^{-10} dx$$

$$= \frac{(x+2)^{-8}}{-8} - 2 \frac{(x+2)^{-9}}{-9} + c = \frac{-1}{8(x+2)^8} + \frac{2}{9(x+2)^9} + c$$

$$366) \int \frac{\cos^4 x - \sin^4 x}{\sqrt{1 + \sin 2x}} dx$$

$$= \int \frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{\sqrt{1 + \sin 2x}} dx = \int \frac{\cos 2x}{\sqrt{1 + \sin 2x}} dx = \int (1 + \sin 2x)^{-\frac{1}{2}} \cos 2x dx$$

$$= \frac{1}{2} \cdot 2 (1 + \sin 2x)^{\frac{1}{2}} + c = \sqrt{1 + \sin 2x} + c$$

$$OR: \int \frac{\cos^4 x - \sin^4 x}{\sqrt{1 + \sin 2x}} dx = \int \frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{\sqrt{\cos^2 x + \sin^2 x + 2 \sin x \cos x}} dx$$

$$= \int \frac{(\cos x + \sin x)(\cos x - \sin x)}{\sqrt{(\cos x + \sin x)^2}} dx = \pm \int \frac{(\cos x + \sin x)(\cos x - \sin x)}{\cos x + \sin x} dx$$

$$= \pm \int (\cos x - \sin x) dx = \pm (\sin x + \cos x) + c$$

$$367) \int \frac{dx}{e^x + 1}$$

$$= \int \frac{1 + e^x - e^x}{e^x + 1} dx = \int \frac{e^x + 1}{e^x + 1} dx - \int \frac{e^x}{e^x + 1} dx = \int dx - \int \frac{e^x}{e^x + 1} dx$$

$$= x - \ln|e^x + 1| + c$$

$$368) \int \sqrt{1 + \cos x} dx$$

$$= \int \sqrt{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}} dx$$

$$= \int \sqrt{2 \cos^2 \frac{x}{2}} dx = \pm \sqrt{2} \int \cos \frac{x}{2} dx = \pm 2\sqrt{2} \sin \frac{x}{2} + c$$

$$369) \int \sqrt{1 - \sin x} dx$$

$$= \int \sqrt{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} - 2 \sin \frac{x}{2} \cos \frac{x}{2}} dx$$

$$= \int \sqrt{\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)^2} dx = \pm \int \left(\cos \frac{x}{2} - \sin \frac{x}{2}\right) dx = \pm 2 \left(\sin \frac{x}{2} + \cos \frac{x}{2}\right) + c$$

$$370) \int \sqrt{1 + \sin x} dx$$

$$= \int \sqrt{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}} dx$$

$$= \int \sqrt{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^2} dx = \pm \int \left(\cos \frac{x}{2} + \sin \frac{x}{2}\right) dx = \pm 2 \left(\sin \frac{x}{2} - \cos \frac{x}{2}\right) + c$$

$$371) \int \sqrt{1 - \cos x} dx$$

$$= \int \sqrt{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}} dx$$

$$= \int \sqrt{2 \sin^2 \frac{x}{2}} dx = \pm \sqrt{2} \int \sin \frac{x}{2} dx = \pm 2\sqrt{2} \cos \frac{x}{2} + c$$

$$372) \int \sqrt{1 - \sin 3x} dx$$

$$= \int \sqrt{\cos^2 \frac{3x}{2} + \sin^2 \frac{3x}{2} - 2 \sin \frac{3x}{2} \cos \frac{3x}{2}} dx$$

$$= \int \sqrt{\left(\cos \frac{3x}{2} - \sin \frac{3x}{2}\right)^2} dx = \pm \int \left(\cos \frac{3x}{2} - \sin \frac{3x}{2}\right) dx$$

$$= \pm \frac{2}{3} \left(\sin \frac{3x}{2} + \cos \frac{3x}{2}\right) + c$$

$$373) \int \frac{\cot x + \tan x}{\cot x - \tan x} dx$$

$$\begin{aligned} &= \int \frac{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}} dx = \int \frac{\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}}{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}} dx = \int \frac{\cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x} dx = \int \frac{1}{\cos^2 x - \sin^2 x} dx \\ &= \int \frac{1}{\cos 2x} dx = \int \sec 2x dx = \frac{1}{2} \ln |\sec 2x + \tan 2x| + c \end{aligned}$$

$$374) \int \sqrt{x^3 - x^2} dx$$

$$\begin{aligned} &= \int \sqrt{x^2(x-1)} dx = \pm \int x \sqrt{x-1} dx = \pm \int (x-1+1) \sqrt{x-1} dx \\ &= \pm \int \left[(x-1)^{\frac{3}{2}} + (x-1)^{\frac{1}{2}} \right] dx = \pm \left[\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} \right] + c \\ &= \pm \left[\frac{2}{5}\sqrt{(x-1)^5} + \frac{2}{3}\sqrt{(x-1)^3} \right] + c \end{aligned}$$

$$375) \int \sqrt[3]{e^x} dx$$

$$= \int (e^x)^{\frac{1}{3}} dx = \int e^{\frac{1}{3}x} dx = 3e^{\frac{1}{3}x} + c = 3\sqrt[3]{e^x} + c$$

$$376) \int \frac{\sec(\ln x) \tan(\ln x)}{x} dx$$

$$= \int \sec(\ln x) \tan(\ln x) \cdot \frac{1}{x} dx$$

$$= \sec(\ln x) + c$$

$$377) \int_1^{36} \frac{1}{\sqrt{x}\sqrt{\sqrt{x}+3}} dx$$

$$= \int_1^{36} \frac{1}{\sqrt{\sqrt{x}+3}} \cdot \frac{1}{\sqrt{x}} dx = \int_1^{36} (\sqrt{x} + 3)^{-\frac{1}{2}} \cdot \frac{1}{\sqrt{x}} dx$$

$$= 2 \int_1^{36} (\sqrt{x} + 3)^{-\frac{1}{2}} \cdot \frac{1}{2\sqrt{x}} dx = 2 \cdot 2 \left[(\sqrt{x} + 3)^{\frac{1}{2}} \right]_1^{36}$$

$$= 4 \left[\sqrt{\sqrt{36} + 3} - \sqrt{\sqrt{1} + 3} \right] = 4[\sqrt{9} - \sqrt{4}] = 4(1) = 4$$

$$378) \int \frac{\csc(\ln x^3) \cot(\ln x^3)}{x} dx$$

$$= \int \csc(\ln x^3) \cot(\ln x^3) \cdot \frac{1}{x} dx$$

$$= -\frac{1}{3} \csc(\ln x^3) + c$$

$$379) \int \frac{A^3}{(A+1)^5} dA$$

$$= \int \left(\frac{A}{A+1} \right)^3 \cdot \frac{1}{(A+1)^2} dA = \frac{1}{4} \left(\frac{A}{A+1} \right)^4 + c$$

$$380) \int (\csc x + \cot x) dx$$

$$= \int \left(\csc x \cdot \frac{\csc x + \cot x}{\csc x + \cot x} + \frac{\cos x}{\sin x} \right) dx = \int \left(\frac{\csc^2 x + \csc x \cot x}{\csc x + \cot x} + \frac{\cos x}{\sin x} \right) dx$$

$$= -\ln|\csc x + \cot x| + \ln|\sin x| + c$$

$$= \ln \left| \frac{\sin x}{\csc x + \cot x} \right| + c$$

$$381) \int \frac{10+15\cos 2x+5\sin 3x+\cos 5x}{32} dx$$

$$\begin{aligned} &= \int \left(\frac{10}{32} + \frac{15}{32} \cos 2x + \frac{5}{32} \sin 3x + \frac{1}{32} \cos 5x \right) dx \\ &= \frac{10}{32} x + \frac{15}{64} \sin 2x - \frac{5}{96} \cos 3x + \frac{1}{160} \sin 5x + c \end{aligned}$$

$$382) \int (1 + \cos^2 3x) dx$$

$$\begin{aligned} &= \int \left(1 + \frac{1}{2} + \frac{1}{2} \cos 6x \right) dx = \int \left(\frac{3}{2} + \frac{1}{2} \cos 6x \right) dx \\ &= \frac{3}{2} x + \frac{1}{12} \sin 6x + c \end{aligned}$$

$$383) \int (\cos^4 x - \sin^4 x) dx$$

$$\begin{aligned} &= \int (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) dx \\ &= \int \cos 2x dx = \frac{1}{2} \sin 2x + c \end{aligned}$$

$$384) \int (1 + \cos 3x)^2 dx$$

$$\begin{aligned} &= \int (1 + 2\cos 3x + \cos^2 3x) dx = \int \left(1 + 2\cos 3x + \frac{1}{2} + \frac{1}{2} \cos 6x \right) dx \\ &= \int \left(\frac{3}{2} + 2\cos 3x + \frac{1}{2} \cos 6x \right) dx = \frac{3}{2} x + \frac{2}{3} \sin 3x + \frac{1}{12} \sin 6x + c \end{aligned}$$

$$385) \int \sin^2 x^2 \cos x^2 x dx$$

$$= \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6} \sin^3 x^2 + c$$

$$386) \int \frac{5}{\csc^2 x \sqrt{\tan x - 5}} dx$$

$$= 5 \int (\tan x - 5)^{-\frac{1}{2}} \sec^2 x dx = 5 \cdot 2 (\tan x - 5)^{\frac{1}{2}} + c$$

$$= 10\sqrt{\tan x - 5} + c$$

$$387) \int \frac{(x-3)^2 + 12x}{x+3} dx$$

$$= \int \frac{x^2 - 6x + 9 + 12x}{x+3} dx = \int \frac{x^2 + 6x + 9}{x+3} dx = \int \frac{(x+3)^2}{x+3} dx$$

$$= \int (x+3) dx = \frac{1}{2}(x+3)^2 + c$$

$$388) \int \frac{nx-m}{\sqrt{nx^2-2mx+k}} dx$$

$$= \int (nx^2 - 2mx + k)^{-\frac{1}{2}} (nx - m) dx$$

$$= \frac{1}{2} \cdot 2 (nx^2 - 2mx + k)^{\frac{1}{2}} + c = \sqrt{nx^2 - 2mx + k} + c$$

$$389) \int (\sin 2x - 1)(\cos^2 2x + 2) dx$$

$$= \int (\cos^2 2x \sin 2x + 2 \sin 2x - \cos^2 2x - 2) dx$$

$$= \int \left(\cos^2 2x \sin 2x + 2 \sin 2x - \frac{1}{2} - \frac{1}{2} \cos 4x - 2 \right) dx$$

$$= \int \left(\cos^2 2x \sin 2x + 2 \sin 2x - \frac{1}{2} \cos 4x - \frac{5}{2} \right) dx$$

$$= -\frac{1}{6} \cos^3 2x - \cos 2x - \frac{1}{8} \sin 4x - \frac{5}{2} x + c$$

$$390) \int \frac{x-\sqrt{x}}{\sqrt{x}} dx$$

$$= \int \frac{\cancel{\sqrt{x}}(\sqrt{x}-1)}{\cancel{\sqrt{x}}} dx = \int (\sqrt{x} - 1) dx = \int \left(x^{\frac{1}{2}} - 1 \right) dx$$

$$= \frac{2}{3} x^{\frac{3}{2}} - x + c = \frac{2}{3} \sqrt{x^3} - x + c$$

$$391) \int \frac{2+\sin x - \sin^2 x + \sin^3 x}{\cos^2 x} dx$$

$$= \int \frac{2+\sin x - \sin^2 x + \sin x - \sin x \cos^2 x}{\cos^2 x} dx = \int \frac{2+2\sin x - \sin^2 x - \sin x \cos^2 x}{\cos^2 x} dx$$

$$= \int (2\sec^2 x + 2\sec x \tan x - \tan^2 x - \sin x) dx$$

$$= \int (2\sec^2 x + 2\sec x \tan x - \sec^2 x + 1 - \sin x) dx$$

$$= \int (\sec^2 x + 2\sec x \tan x + 1 - \sin x) dx$$

$$= \tan x + 2\sec x + x + \cos x + c$$

$$392) \int \sin \left(\frac{x-1}{x+1} \right) \left(\frac{3}{x+1} \right)^2 dx$$

$$= 9 \int \sin \left(\frac{x-1}{x+1} \right) \frac{1}{(x+1)^2} dx = -\frac{9}{2} \cos \left(\frac{x-1}{x+1} \right) + c$$

$$393) \int \frac{\sec^2(\sin x)}{\cos x} dx$$

$$= \int \sec^2(\sin x) \cos x dx = \tan(\sin x) + c$$

$$394) \int \left(\frac{\sin x}{1-\cos x} + \cot x \right) dx$$

$$\begin{aligned} &= \int \left(\frac{\sin x}{1-\cos x} \cdot \frac{1+\cos x}{1+\cos x} + \frac{\cos x}{\sin x} \right) dx = \int \left(\frac{\sin x(1+\cos x)}{1-\cos^2 x} + \frac{\cos x}{\sin x} \right) dx \\ &= \int \left(\frac{\sin x(1+\cos x)}{\sin^2 x} + \frac{\cos x}{\sin x} \right) dx = \int \left(\frac{1+2\cos x}{\sin x} \right) dx = \int \left(\csc x + 2 \frac{\cos x}{\sin x} \right) dx \\ &= -\ln|\csc x + \cot x| + 2 \ln|\sin x| + c = \ln \left| \frac{\sin^2 x}{\csc x + \cot x} \right| + c \end{aligned}$$

$$395) \int_4^9 \frac{dx}{x-\sqrt{x}} dx$$

$$\begin{aligned} &= \int_4^9 \frac{dx}{\sqrt{x}(\sqrt{x}-1)} dx = 2 \left[\ln|\sqrt{x}-1| \right]_4^9 = 2[(\ln 3 - 1) - (\ln 2 - 1)] \\ &= 2[\ln 2 - \ln 1] = 2 \ln 2 = \ln 4 \end{aligned}$$

$$396) \int \frac{\sqrt{1+\sin x}}{\sin x} dx$$

$$\begin{aligned} &= \int \frac{\sqrt{\sin^2 \frac{x}{2} + 2\sin \frac{x}{2} \cos \frac{x}{2} + \cos^2 \frac{x}{2}}}{\sin x} dx = \int \frac{\sqrt{(\sin \frac{x}{2} + \cos \frac{x}{2})^2}}{\sin x} dx \\ &= \pm \int \frac{\sin \frac{x}{2} + \cos \frac{x}{2}}{2\sin \frac{x}{2} \cos \frac{x}{2}} dx = \pm \frac{1}{2} \int \left(\frac{1}{\cos \frac{x}{2}} + \frac{1}{\sin \frac{x}{2}} \right) dx = \pm \frac{1}{2} \int \left(\sec \frac{x}{2} + \csc \frac{x}{2} \right) dx \\ &= \pm \left[\ln \left| \sec \frac{x}{2} + \tan \frac{x}{2} \right| - \ln \left| \csc \frac{x}{2} + \cot \frac{x}{2} \right| \right] + c \end{aligned}$$

$$397) \int \frac{1-\cos 2x}{\tan x} dx$$

$$\begin{aligned} &= \int \frac{1-1+2\sin^2 x}{\tan x} dx = \int 2\sin^2 x \cdot \cot x dx = 2 \int \sin^2 x \cdot \frac{\cos x}{\sin x} dx \\ &= 2 \int \sin x \cos x dx = 2 \cdot \frac{1}{2} \sin^2 x + c = \sin^2 x + c \end{aligned}$$

$$398) \int \frac{1-\cos 2x}{1-\sin 2x} dx$$

$$= \int \frac{1-\cos 2x}{1-\sin 2x} \cdot \frac{1+\sin 2x}{1+\sin 2x} dx = \int \frac{1+\sin 2x - \cos 2x - \cos 2x \sin 2x}{1-\sin^2 x} dx$$

$$= \int \frac{1+\sin 2x - \cos 2x - \cos 2x \sin 2x}{\cos^2 2x} dx$$

$$= \int \left(\sec^2 2x + \sec 2x \tan 2x - \sec 2x - \frac{\sin 2x}{\cos 2x} \right) dx$$

$$= \frac{1}{2} [\tan 2x + \sec 2x - \ln |\sec 2x + \tan 2x| + \ln |\cos 2x|] + c$$

$$399) \int \frac{x^3}{(x-1)^5} dx$$

$$= \int \left(\frac{x}{x-1} \right)^3 \frac{1}{(x-1)^2} dx = -\frac{1}{4} \left(\frac{x}{x-1} \right)^3 + c$$

$$400) \int \left(\frac{\cos x}{1-\sin x} - \frac{\cos x}{1+\sin x} \right) dx$$

$$= \int \frac{\cos x + \cos x \sin x - \cos x + \cos x \sin x}{1-\sin^2 x} dx$$

$$= \int \frac{2\cos x \sin x}{\cos^2 x} dx = 2 \int \frac{\sin x}{\cos x} dx = -2 \ln |\cos x| + c$$

$$401) \int \left(\frac{\cos x}{1-\sin x} + \frac{\cos x}{1+\sin x} \right) dx$$

$$= \int \frac{\cos x + \cos x \sin x + \cos x - \cos x \sin x}{1-\sin^2 x} dx$$

$$= \int \frac{2\cos x}{\cos^2 x} dx = 2 \int \frac{1}{\cos x} dx = 2 \int \sec x dx = 2 \ln |\sec x + \tan x| + c$$

$$402) \int \frac{1}{1+\sec x} dx$$

$$= \int \frac{1}{1+\frac{1}{\cos x}} dx = \int \frac{\cos x}{1+\cos x} \cdot \frac{1-\cos x}{1-\cos x} dx$$

$$= \int \frac{\cos x - \cos^2 x}{\sin^2 x} dx = \int (\csc x \cot x - \cot^2 x) dx$$

$$= \int (\csc x \cot x - \csc^2 x + 1) dx$$

$$= -\csc x + \cot x + x + c$$

$$403) \int \frac{1}{1+\csc x} dx$$

$$= \int \frac{\sin x}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x} dx = \int \frac{\sin x - \sin^2 x}{1-\sin^2 x} dx = \int \frac{\sin x - \sin^2 x}{\cos^2 x} dx$$

$$= \int (\sec x \tan x - \tan^2 x) dx$$

$$= \int (\sec x \tan x - \sec^2 x + 1) dx = \sec x - \tan x + x + c$$

$$404) \int \left(1 - \frac{\cos^2 x}{1+\sin x} \right) dx$$

$$= \int \left(1 - \frac{(1-\sin x)(1+\sin x)}{1+\sin x} \right) dx = \int (1 - 1 + \sin x) dx$$

$$= \int \sin x dx = -\cos x + c$$

$$405) \int \frac{\cot^2 x}{1+\sin x} dx$$

$$= \int \frac{\cot^2 x}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x} dx = \int \frac{\cot^2 x - \sin x \cot^2 x}{1-\sin^2 x} dx$$

$$= \int \left(\frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} \right) dx = \int (\csc^2 x - \csc x) dx$$

$$= -\cot x + \ln|\csc x + \cot x| + c$$

$$406) \int \left(\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \right) dx$$

$$\begin{aligned} &= \int \left(\frac{\cos x}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x} + \frac{1+\sin x}{\cos x} \right) dx = \int \left(\frac{\cos x(1-\sin x)}{\cos^2 x} + \frac{1+\sin x}{\cos x} \right) dx \\ &= \int \frac{1-\sin x+1+\sin x}{\cos x} dx = \int \frac{2}{\cos x} dx = 2 \int \sec x dx \\ &= 2 \ln |\sec x + \tan x| + c \end{aligned}$$

$$407) \int \frac{e^{(x^2+2\ln x)-xe^{x^2}}}{x-1} dx$$

$$\begin{aligned} &= \int \frac{e^{x^2} e^{\ln x^2 - xe^{x^2}}}{x-1} dx = \int \frac{e^{x^2} x^2 - x xe^{x^2}}{x-1} dx = \int \frac{xe^{x^2}(x-1)}{x-1} dx \\ &= \int e^{x^2} x dx = \frac{1}{2} e^{x^2} + c \end{aligned}$$

$$408) \int \frac{\cos 2x}{1-\cos 4x} dx$$

$$\begin{aligned} &= \int \frac{\cos 2x}{1-1+2\sin^2 2x} dx = \int \frac{\cos 2x}{2\sin^2 2x} dx \\ &= \frac{1}{2} \int \csc 2x \cot 2x dx = -\frac{1}{4} \csc 2x + c \end{aligned}$$

$$409) \int e^x \sqrt{a-be^x} dx$$

$$= \int e^x (a-be^x)^{\frac{1}{2}} dx = -\frac{1}{b} \cdot \frac{2}{3} (a-be^x)^{\frac{3}{2}} + c = -\frac{2}{3b} \sqrt{(a-be^x)^3} + c$$

$$410) \int \frac{(x-1)^5}{(x+1)^7} dx$$

$$= \int \left(\frac{x-1}{x+1}\right)^5 \frac{1}{(x+1)^2} dx = \frac{1}{12} \left(\frac{x-1}{x+1}\right)^6 + c$$

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